Sequences of Challenges to Build Mathematical Problem-Solvers, KS2

Show the thinking of expert problem-solvers to give all children success. Build the pre-skills to access tasks, focus thinking on specific skills, use scaffolds, open up challenges.

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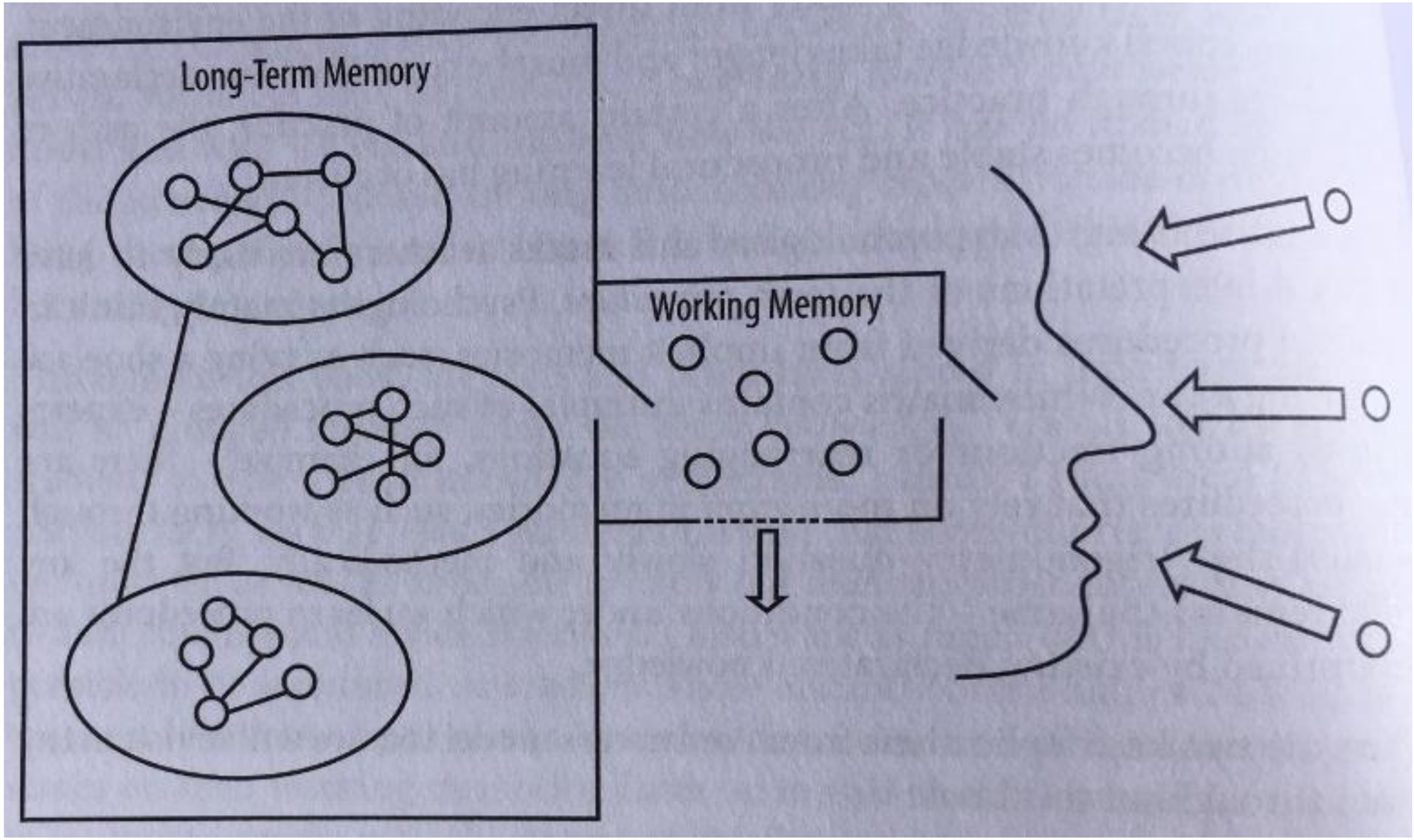
Y3 → Y6



High cognitive demand

Connecting learning to schemas

When students are presented with a mathematics word problem, their first response often is to try to compute an answer, even before they have tried to understand the problem.

Studies of expertise have shown that experts attend more to the underlying structure of a problem, whereas novices rely more on surface features.

Expert problem solvers typically spend more time thinking about problems and trying to understand them than do novices, who tend to immediately execute a solution.

Removing Opportunities to Calculate Improves Students' Performance on Subsequent Word Problems. *Givvin and Stigler (2019)*

Sandcastles

Lisa went on holiday.

In 5 days she made 80 sandcastles.

Each day she made 4 fewer castles than the day before.

How many castles did she make each day?

Lisa went on making 4 fewer castles each day.

How many castles did she make altogether?

$$80 \div 5 = 16$$

24 20 16 12 8

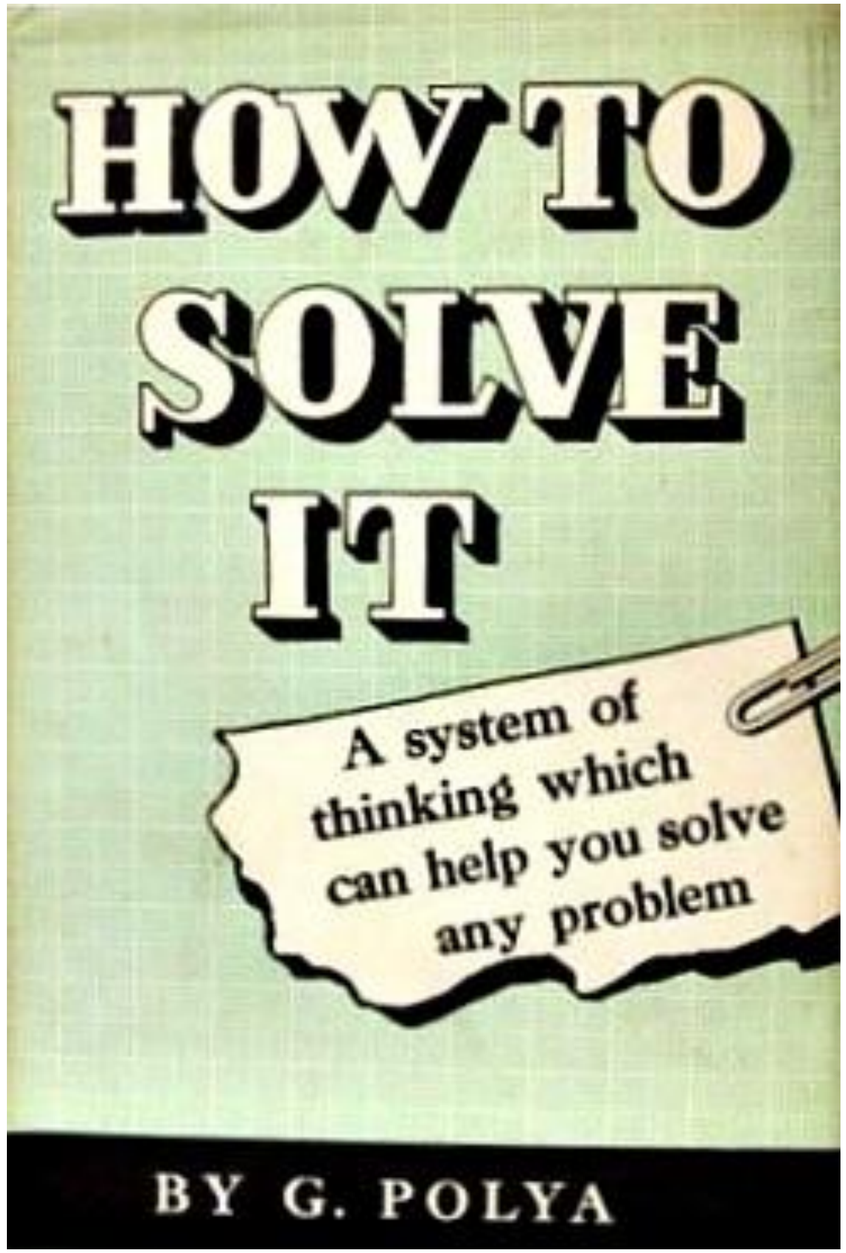
Money bags

Ram divided 15 pennies among four small bags.

He could then pay any sum of money from 1p to 15p, without opening any bag.

How many pennies did Ram put in each bag?

1p 2p 4p 8p



Understand the Problem

Devise a Plan

Carry Out the Plan

Review and Extend

Referring to the importance of the knowledge base:

Expertise in various domains depends on having access to some 50,000 chunks of knowledge in long-term memory.

A lot of what appears to be strategy use is in fact reliance on well developed knowledge chunks.

Nonetheless, it is important not to overplay the roles of these knowledge schemata…

…mathematicians also employ a wide range of strategies when confronted with problems beyond the routine.

Learning to think mathematically: problem-solving, metacognition and sense-making in mathematics. *Alan Schoenfeld*

The sum of four whole numbers is 23.

The difference between the smallest and the largest number is 6.

All four numbers are different.

What could the four numbers be?

*Find **all the possible answers** to this question.*

4 10 7

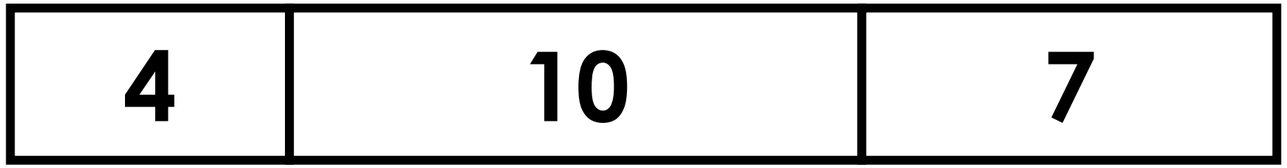
4 10 7

The sum of these numbers…

The difference between the largest and smallest number is…

4 10 7

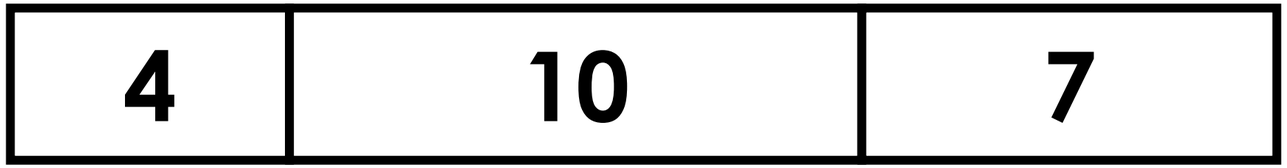
The sum of these numbers…



The difference between the largest and smallest number is…

4 10 7

The sum of these numbers…



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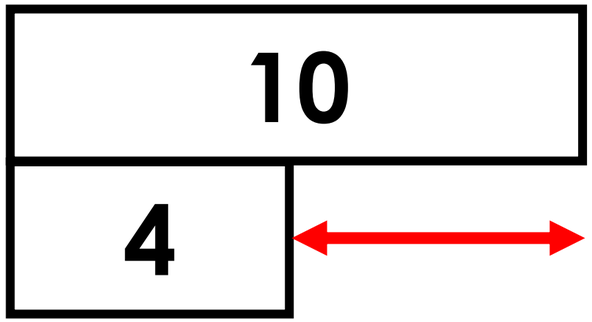
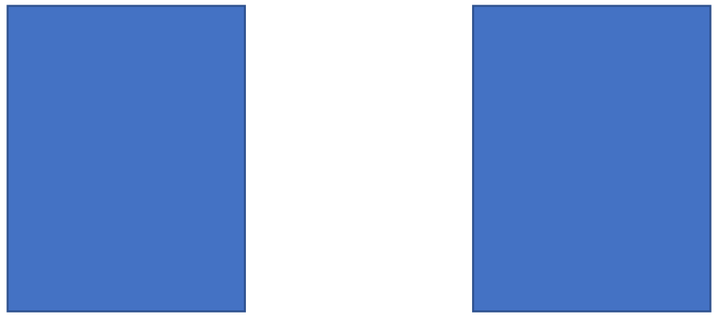


3 6 8 7

The sum of these numbers…

The difference between the largest and smallest number is…

3  7

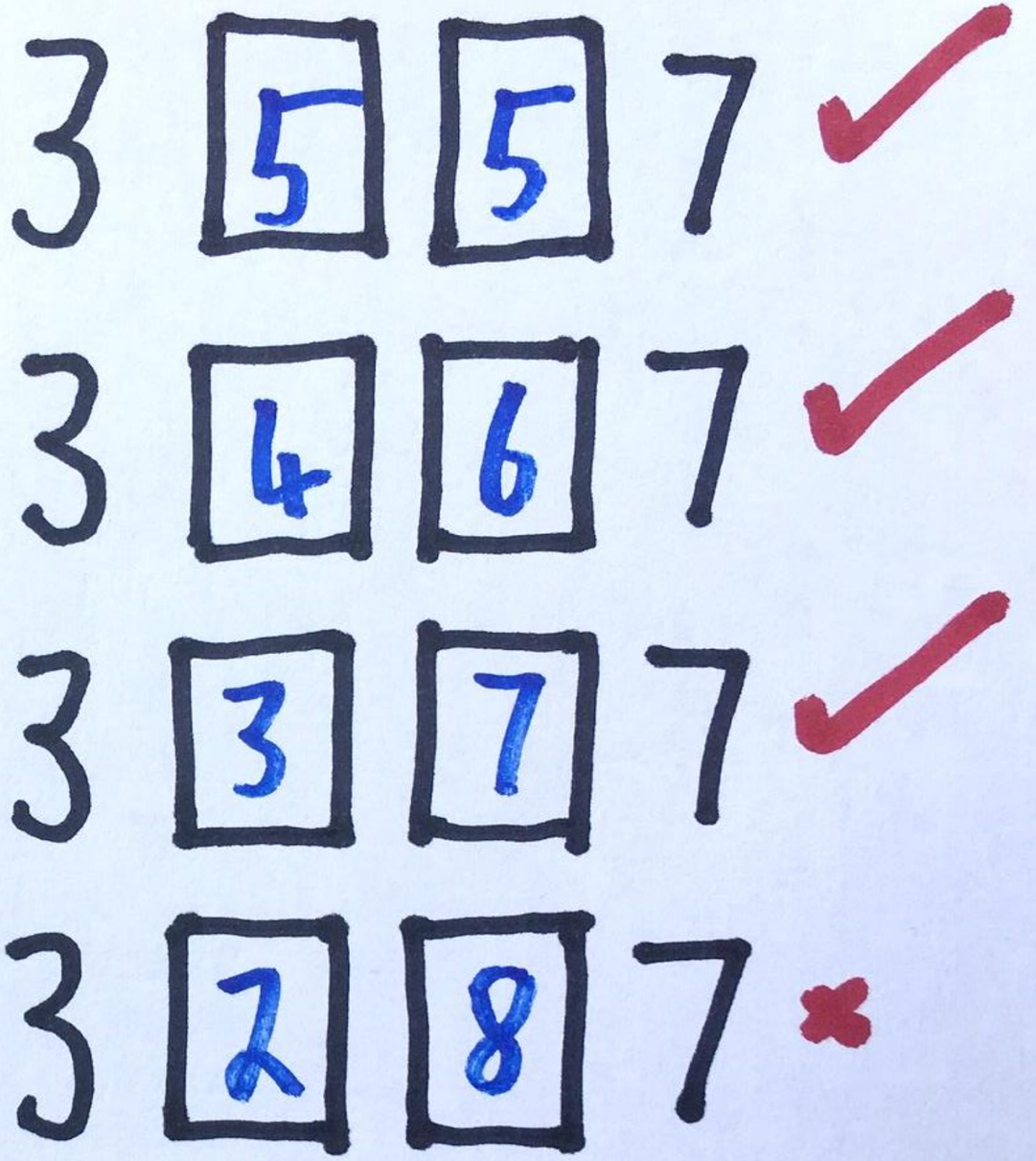
The sum of these numbers is **20**

The difference between the largest and smallest number is **4**

Sum = 20

Diff = 4

3 ways



The sum of these numbers is **19**

The difference between the largest and smallest number is **7**

2 8 9

The sum of these numbers is **19**

The difference between the largest and smallest number is **7**

The sum of four whole numbers is 23.

The difference between the smallest and the largest number is 6.

All four numbers are different.

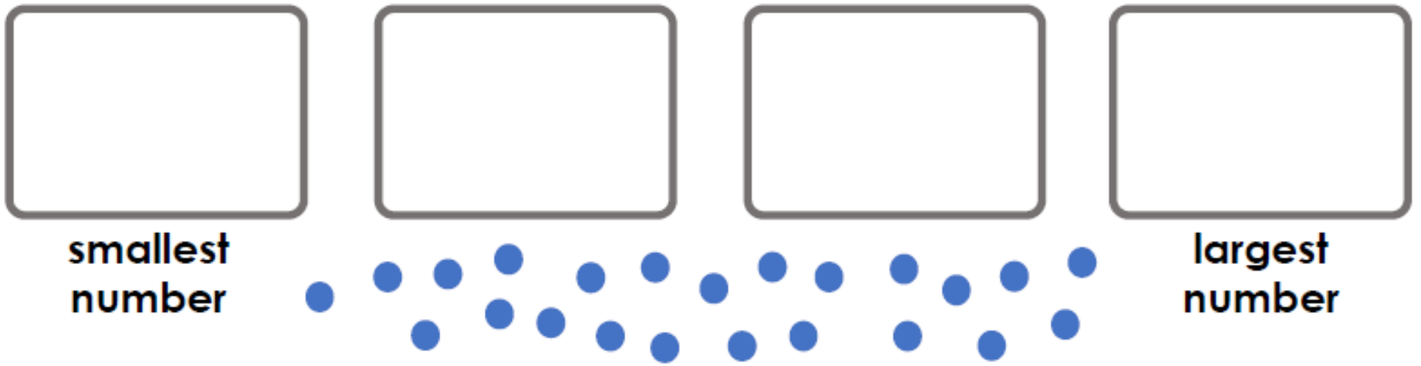
What could the four numbers be?

*Find **all the possible answers** to this question.*

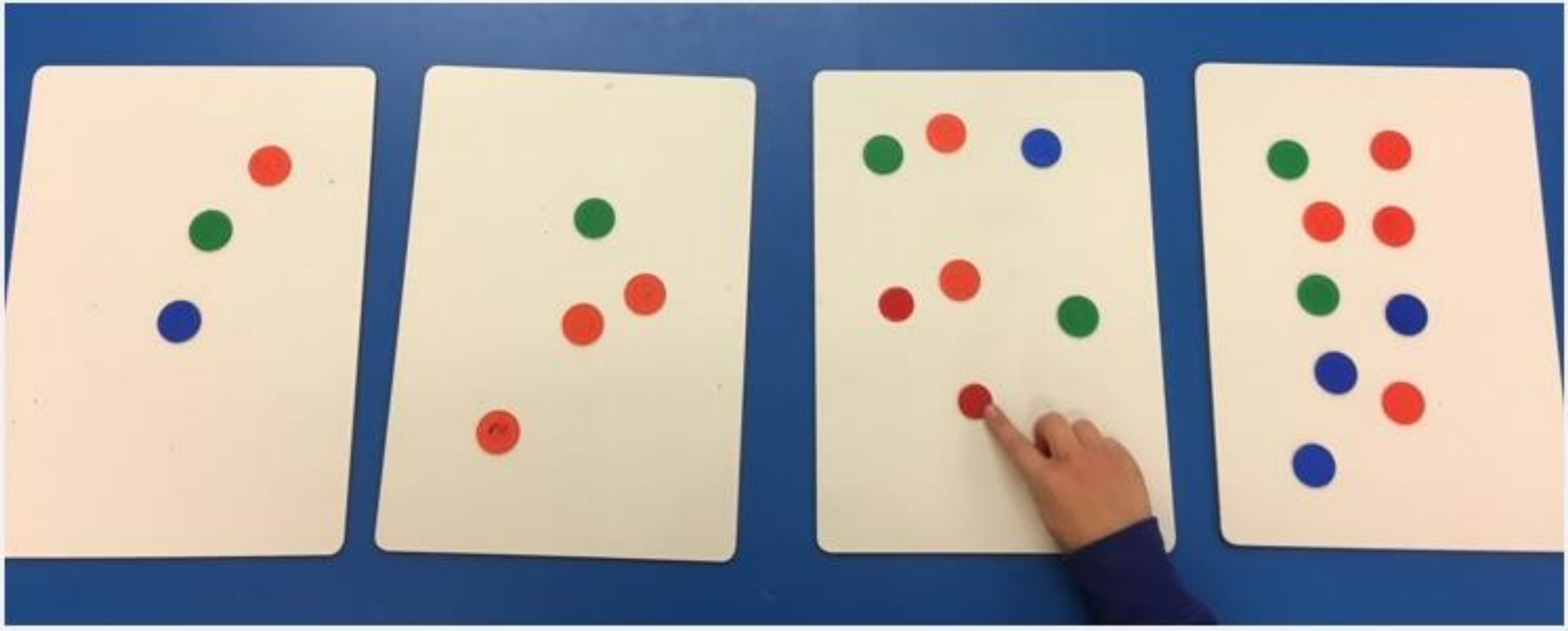
SUPPORT

Tip: Share 23 counters between four whiteboards.

Each whiteboard represents one of the numbers.



***Remember:** the difference between the smallest number and the largest number is 6.*



The sum of four whole numbers is 23.

The difference between the smallest and the largest number is 6.

All four numbers are different.

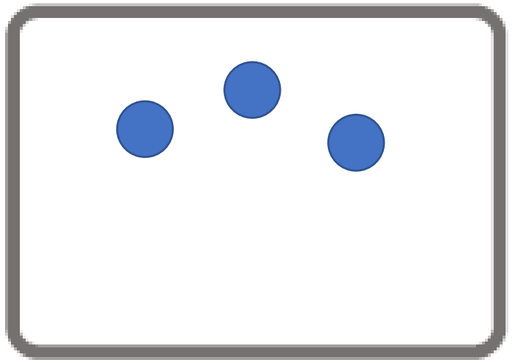
What could the four numbers be?

*Find **all the possible answers** to this question.*

23 ÷ 4 = 5 r 3

'Start with the smallest and largest numbers'

Spot the mistake:



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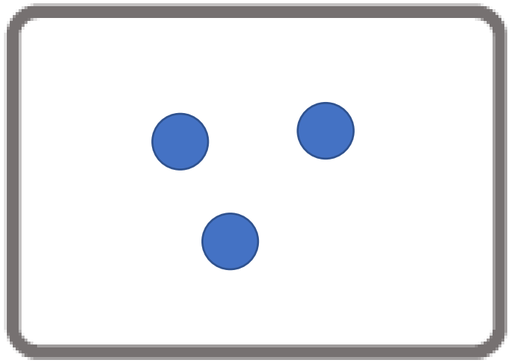

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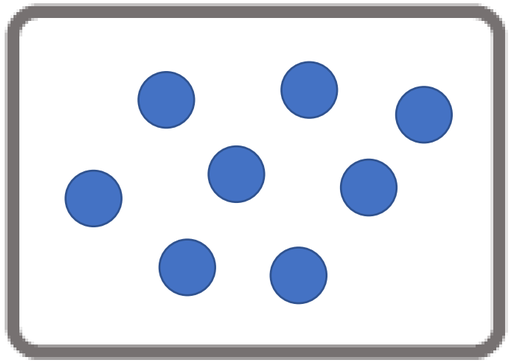

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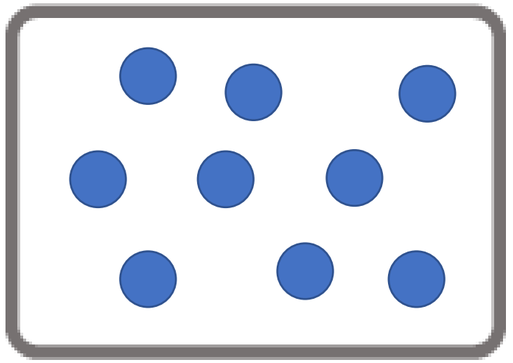

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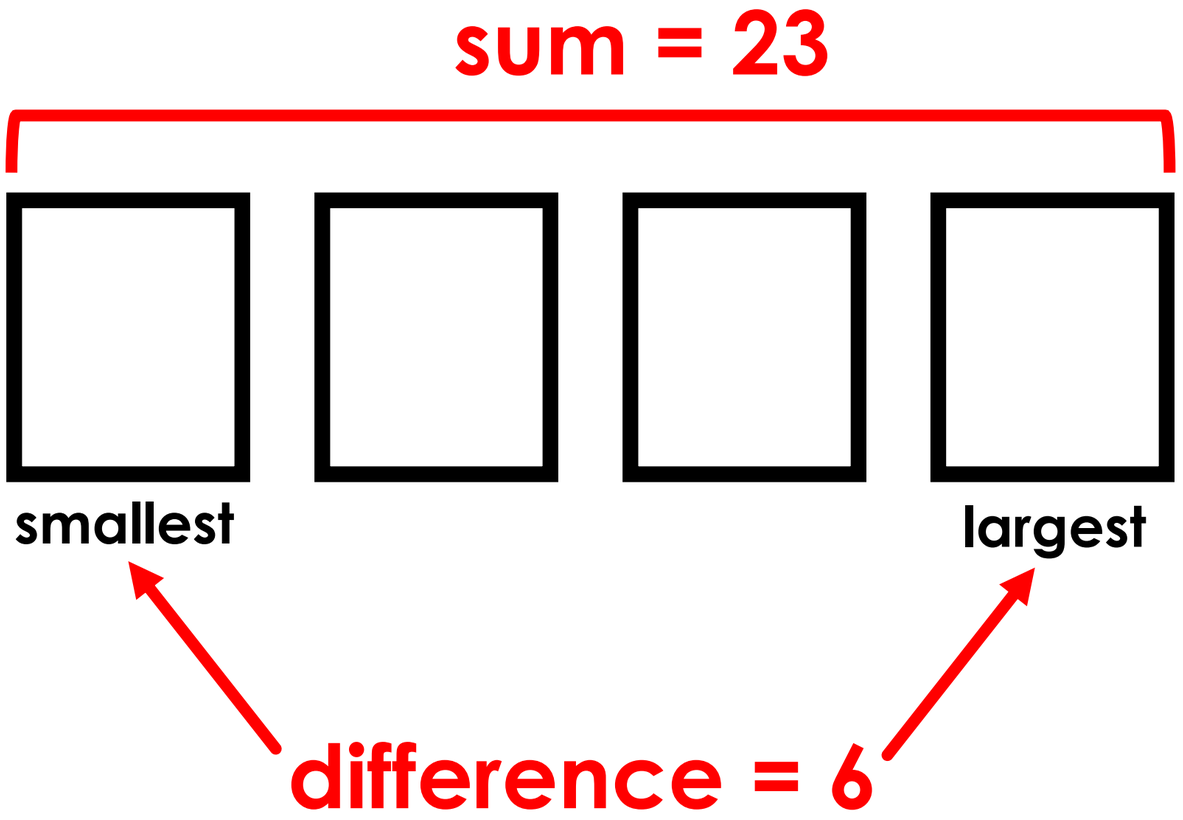
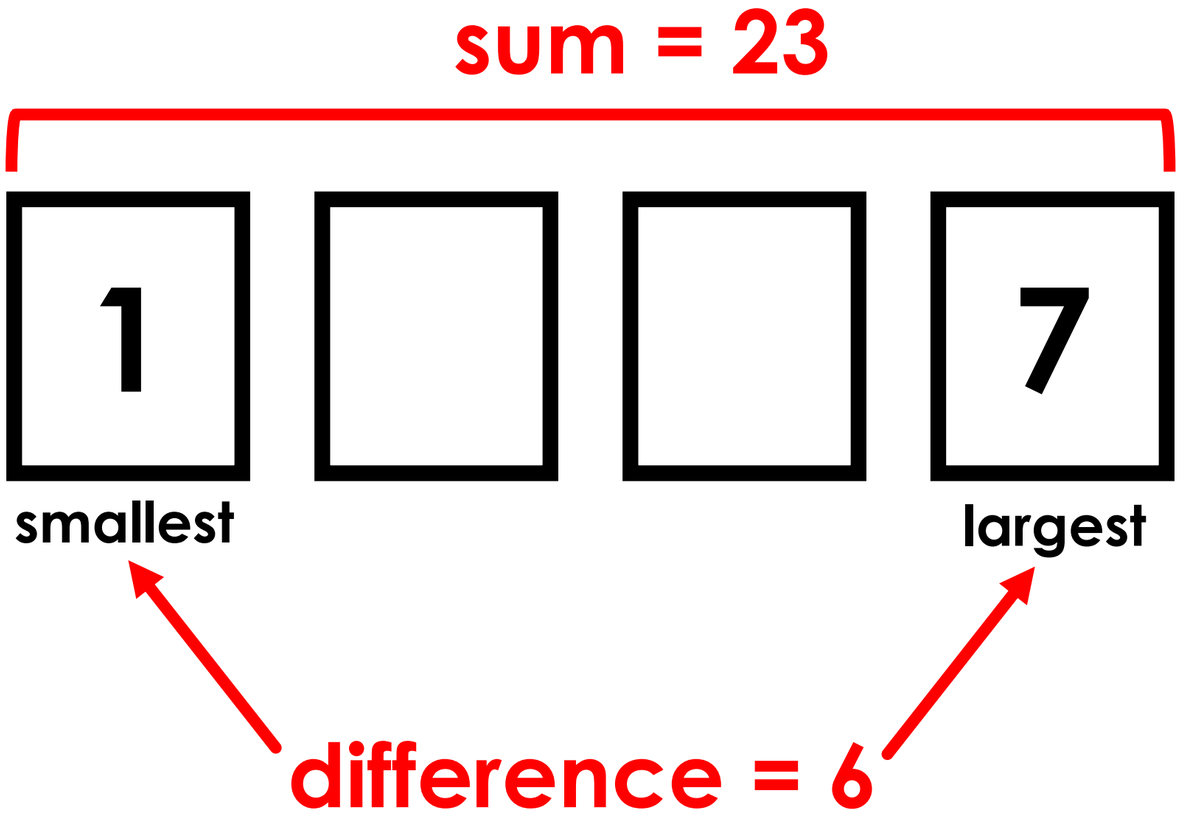
Four numbers challenge



Four numbers challenge



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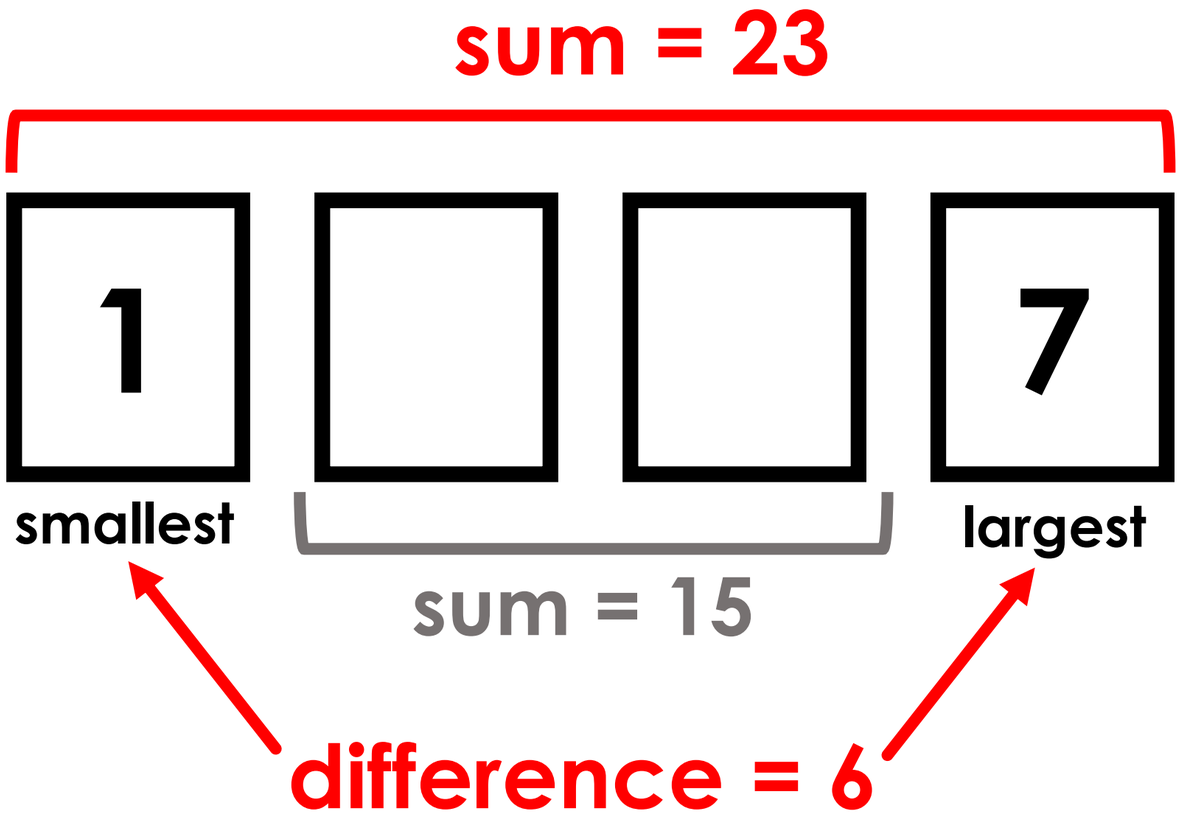
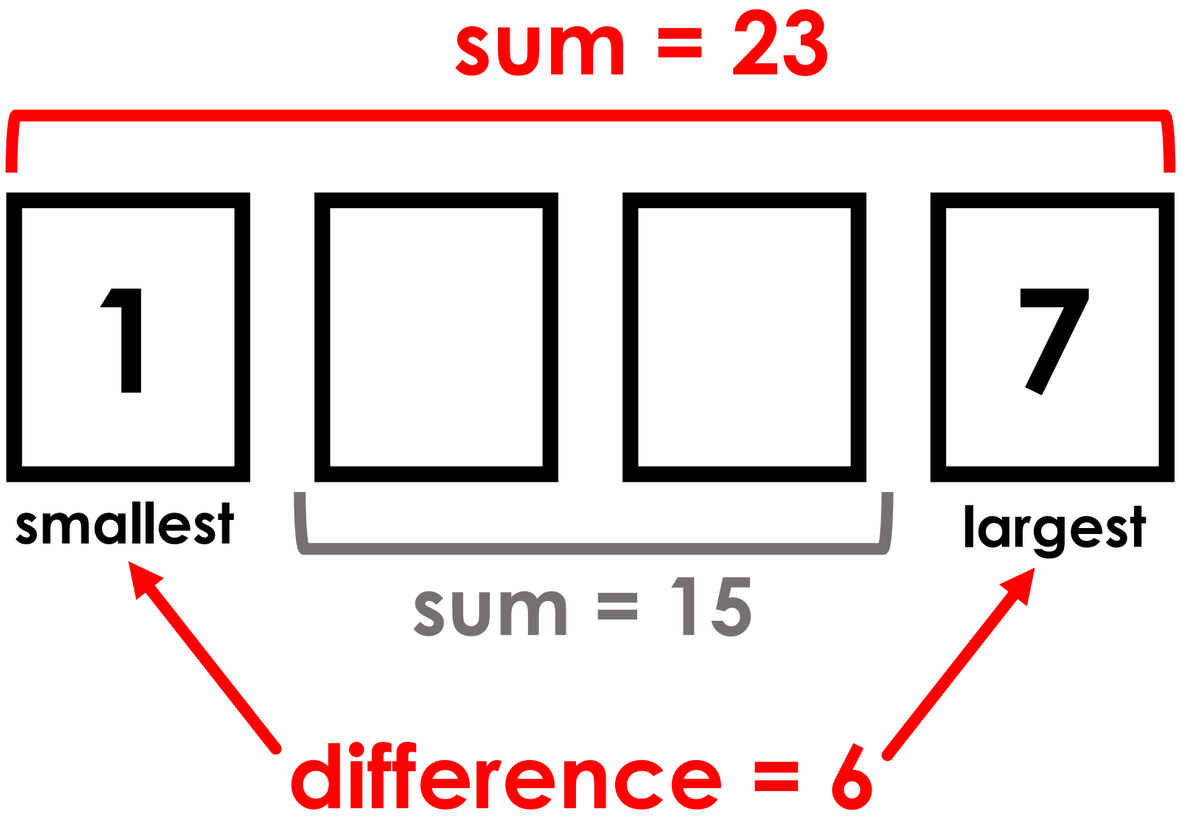
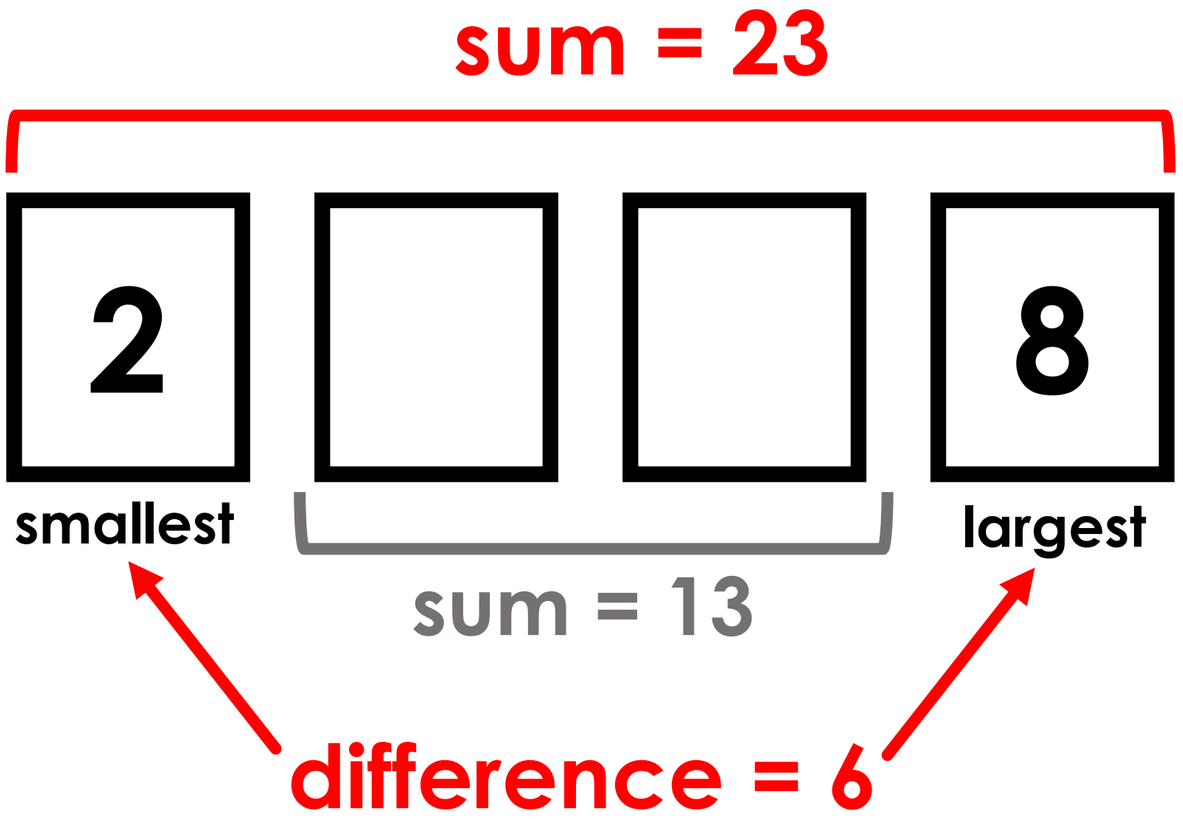


Four numbers challenge



Not possible with two whole numbers less than 7

Four numbers challenge



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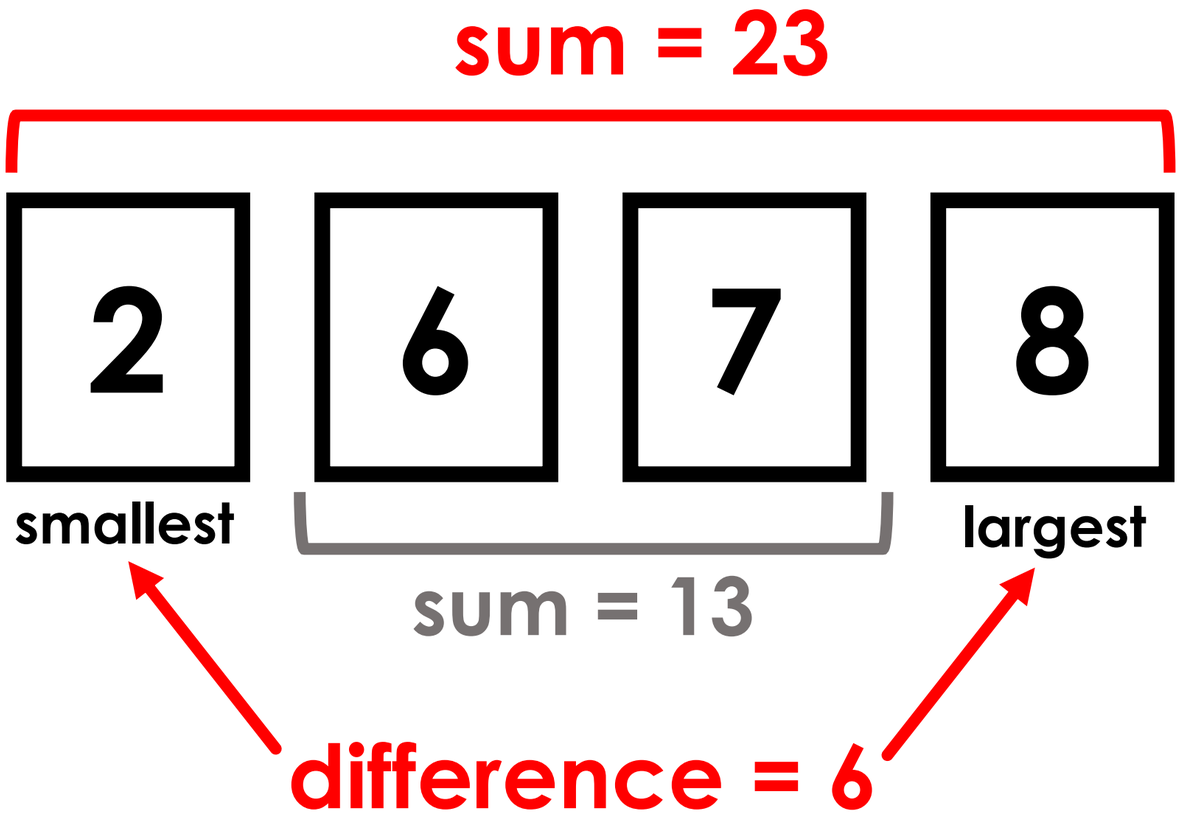

Answers

2, 6, 7, 8

EXPLAIN

Explain how you know that this statement is correct:

'The largest number must be more than 7'

EXTEND

The sum of four numbers is 25. All four numbers are different.

The difference between the smallest and the largest number is 4.

All four numbers are multiples of 0.5

What could the four numbers be?

*Find **all the possible answers** to this question.*

One answer: 4, 5.5, 7.5, 8

A bat and a ball cost £8.

The bat costs £2 more than the ball.

Bat =

Ball =

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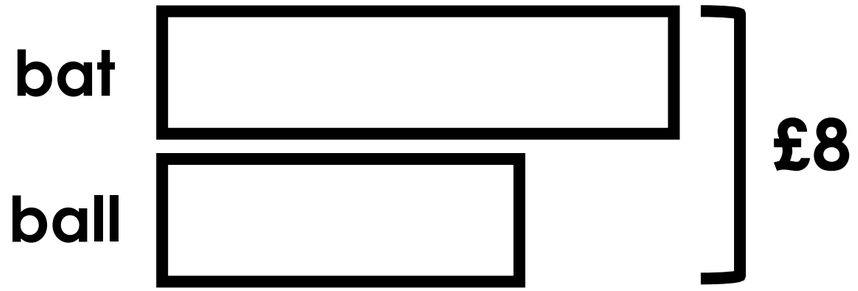

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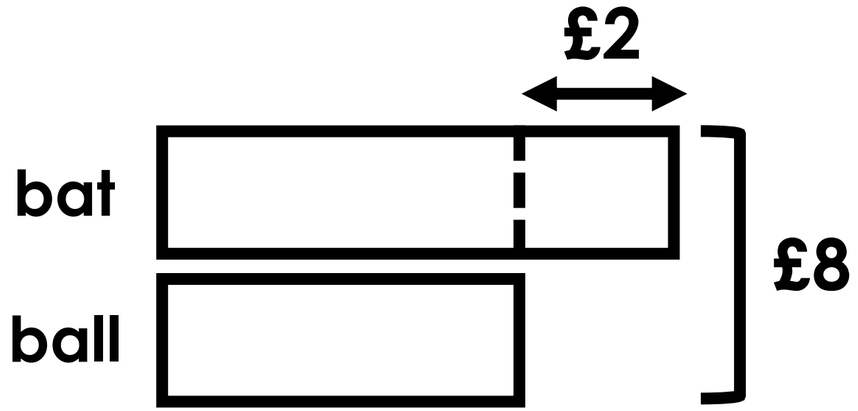

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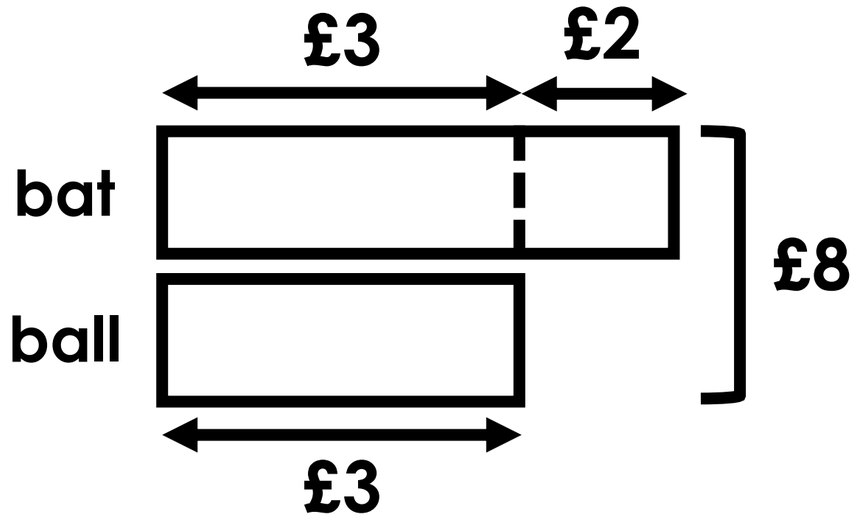

A bat and a ball cost £8.

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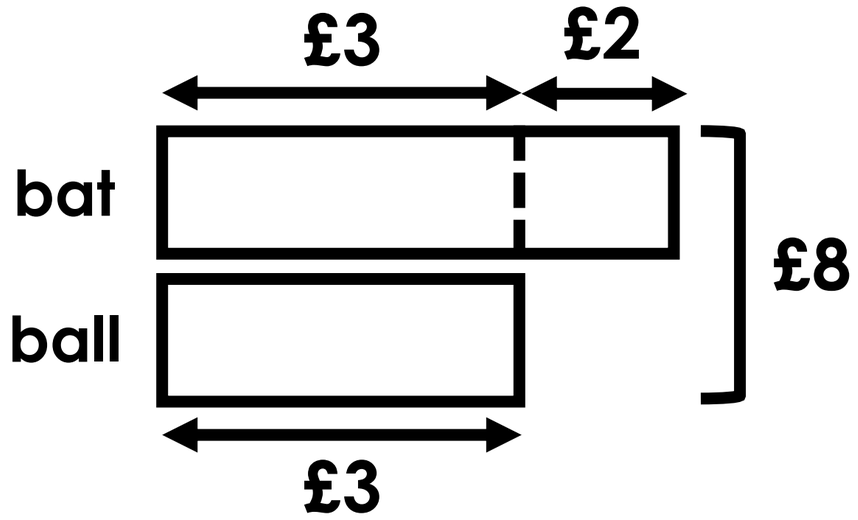
Bat = £5

Ball = £3



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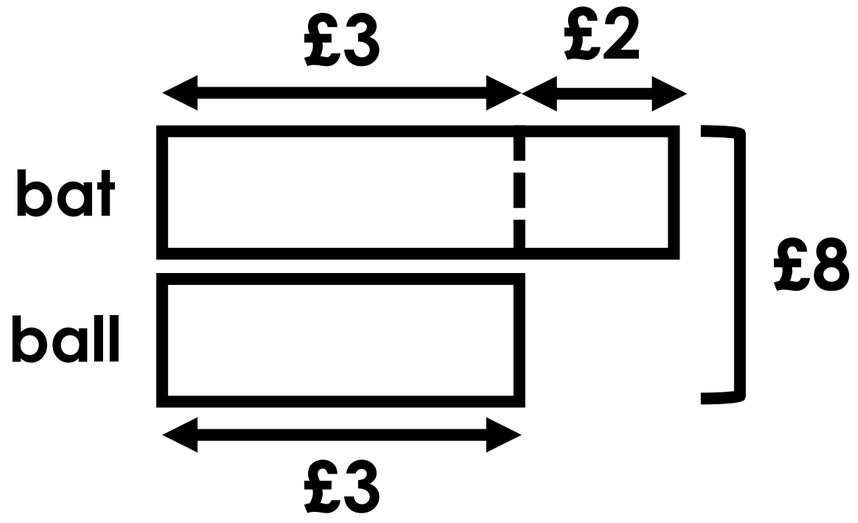
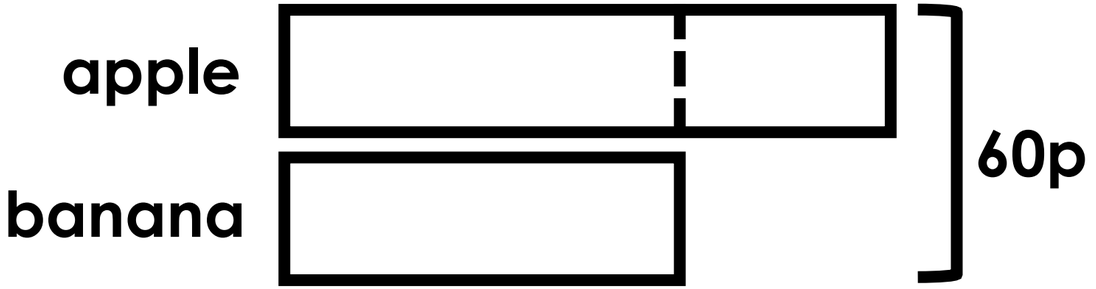


An apple and a banana cost 60p.

An apple costs 10p more than a banana.

How much does an apple cost?

Apple =



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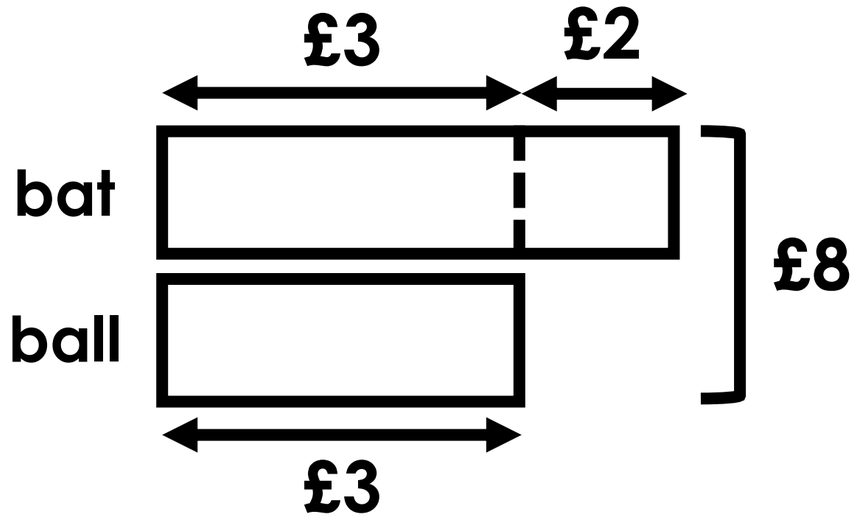
Ball = £3



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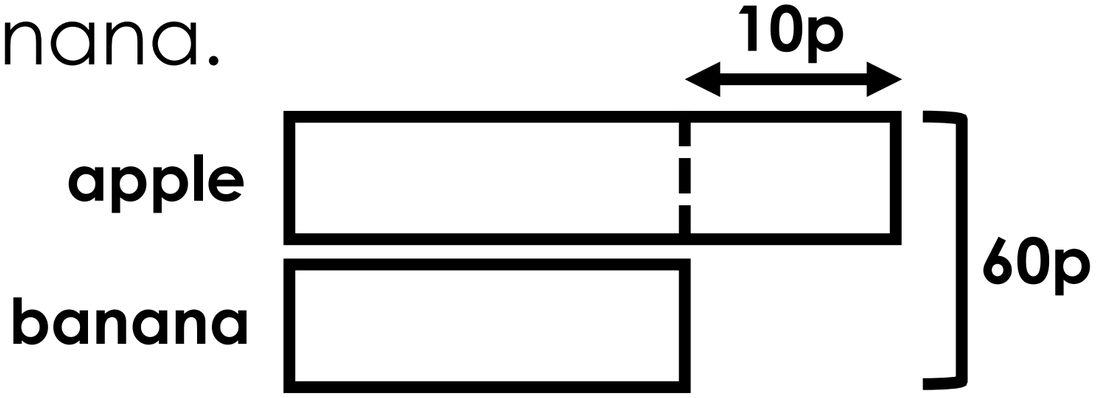

A bat and a ball cost £8.

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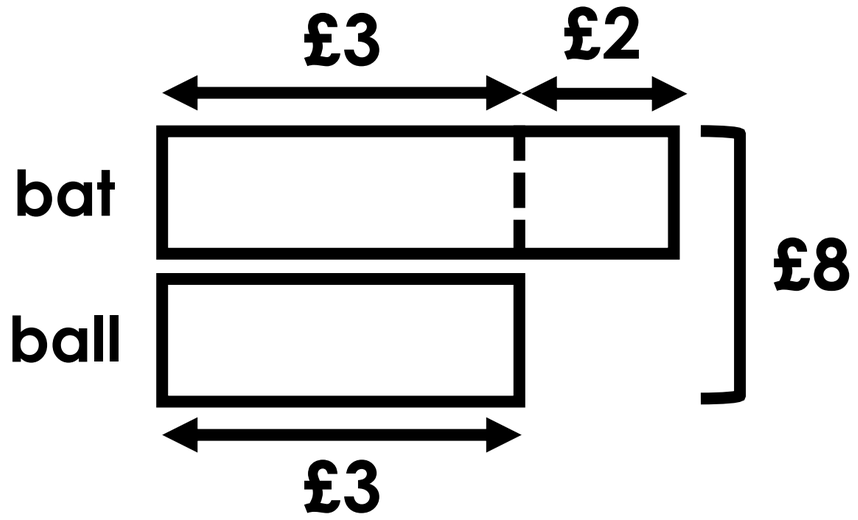
Bat = £5

Ball = £3



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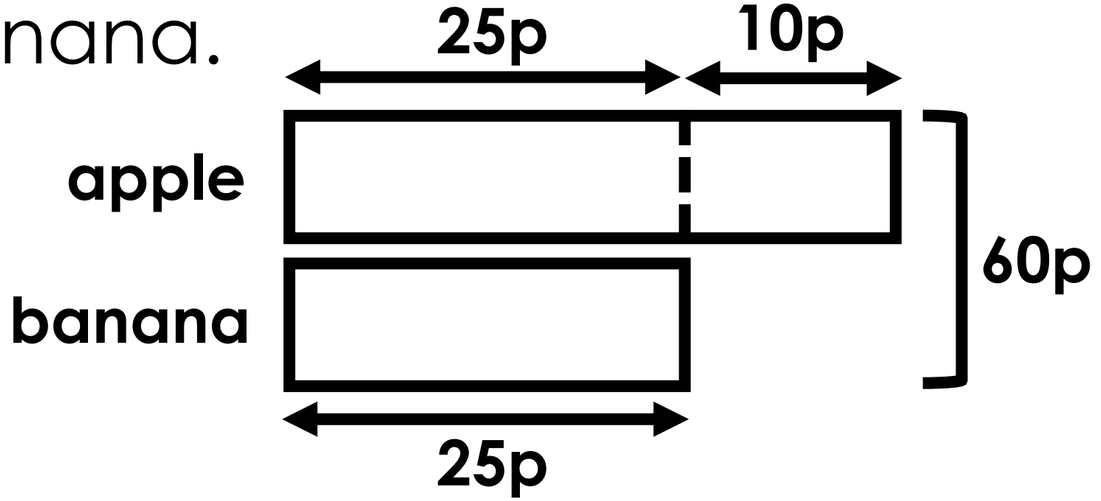
An apple costs 10p more than a banana.

How much does an apple cost?

Apple =



A bat and a ball cost £8.

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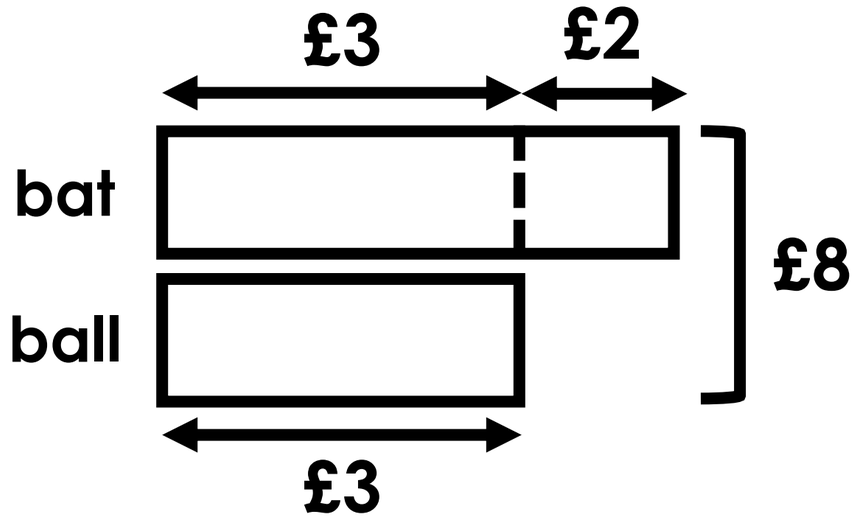
Bat = £5

Ball = £3

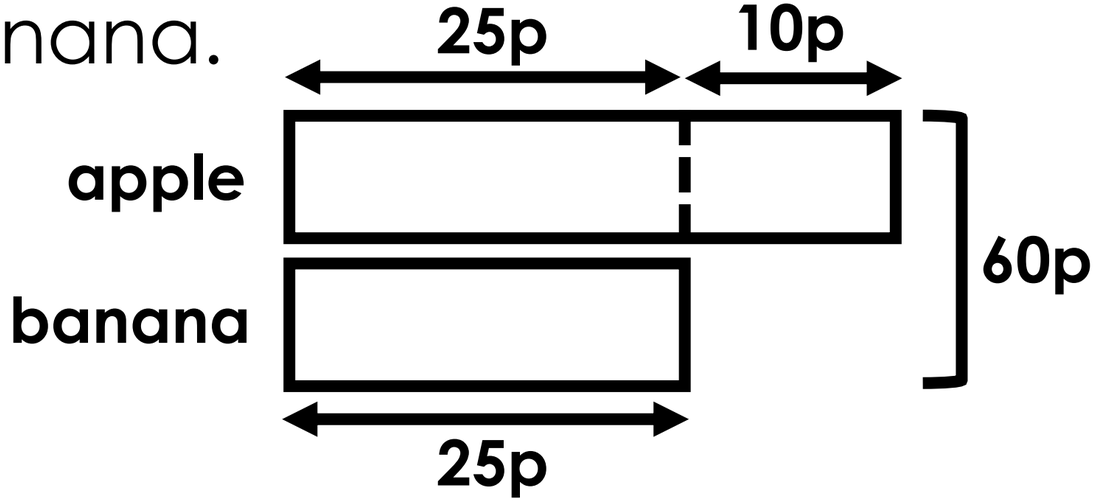


An apple and a banana cost 60p.

An apple costs 10p more than a banana.

How much does an apple cost?

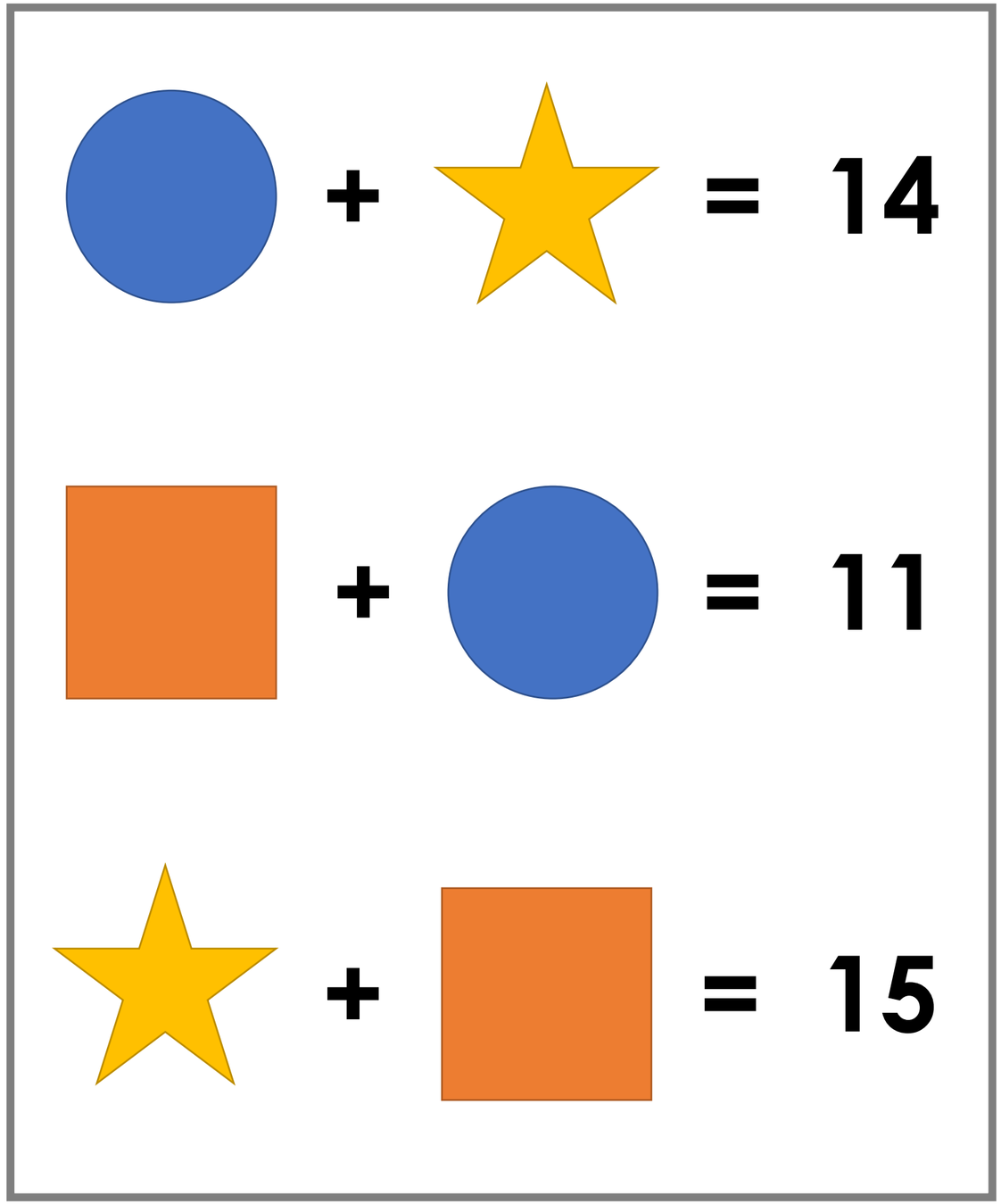
Apple = 35p



The circle is worth…

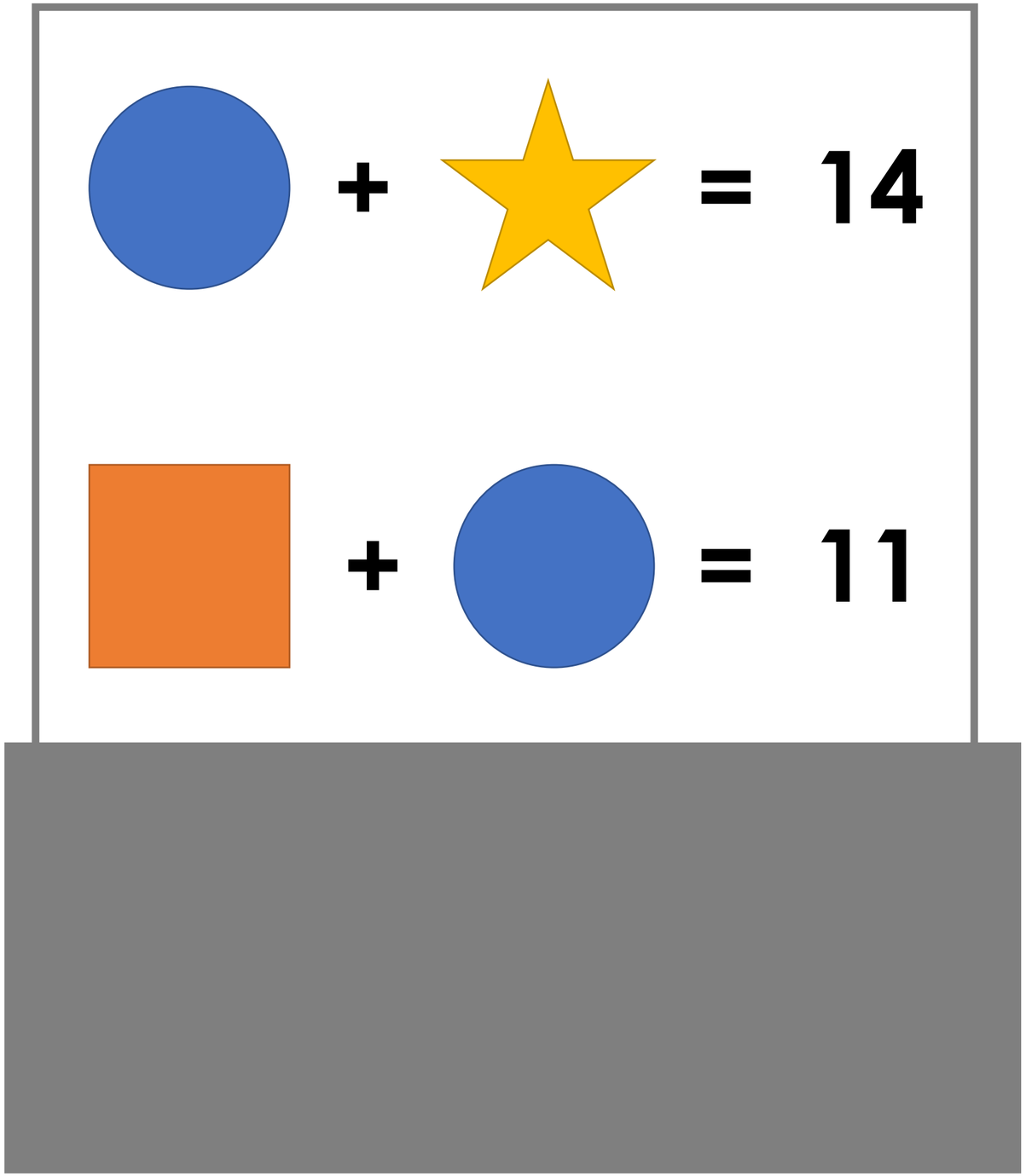
The circle could be worth…

Which shape is worth more?



largest

smallest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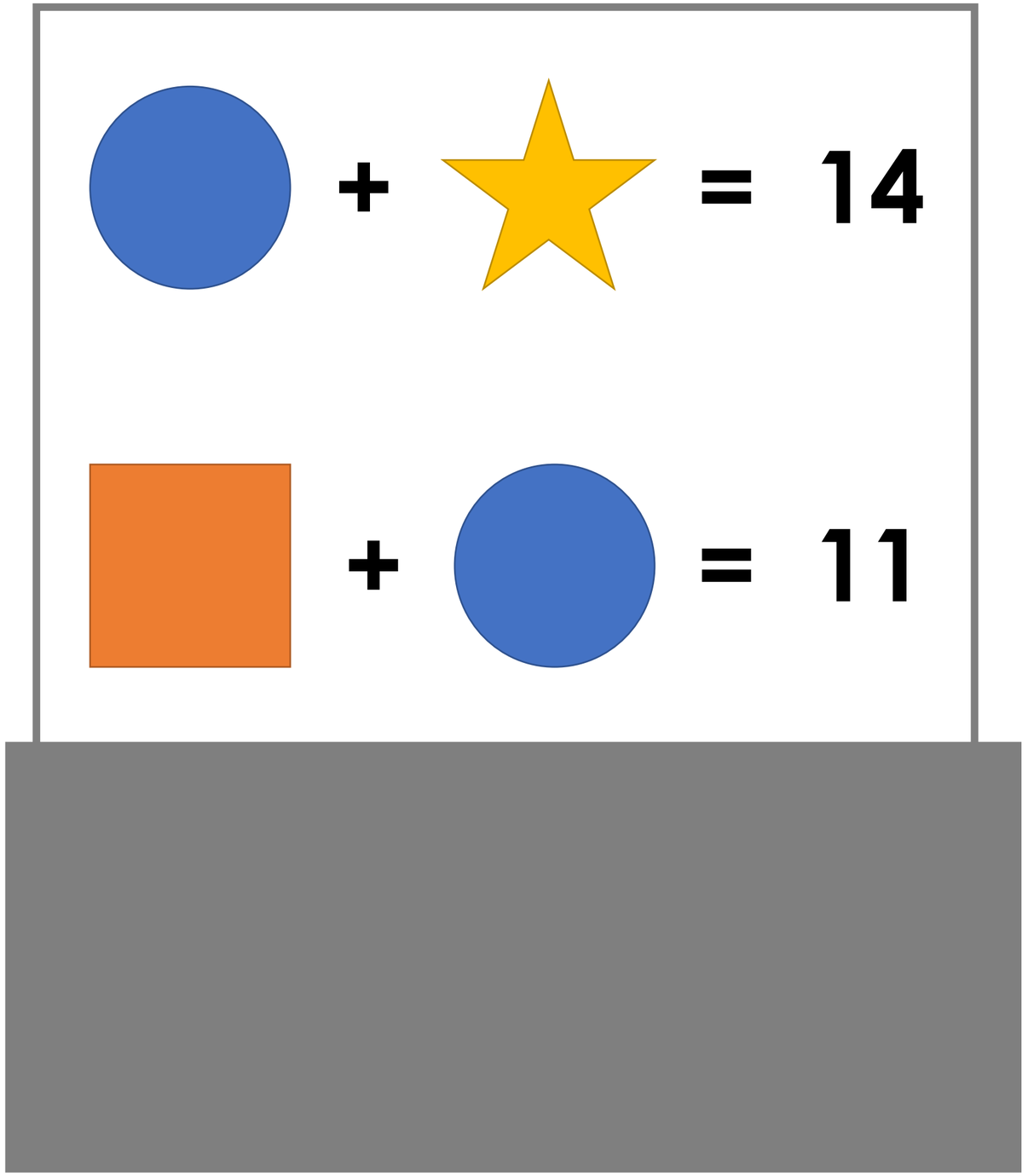
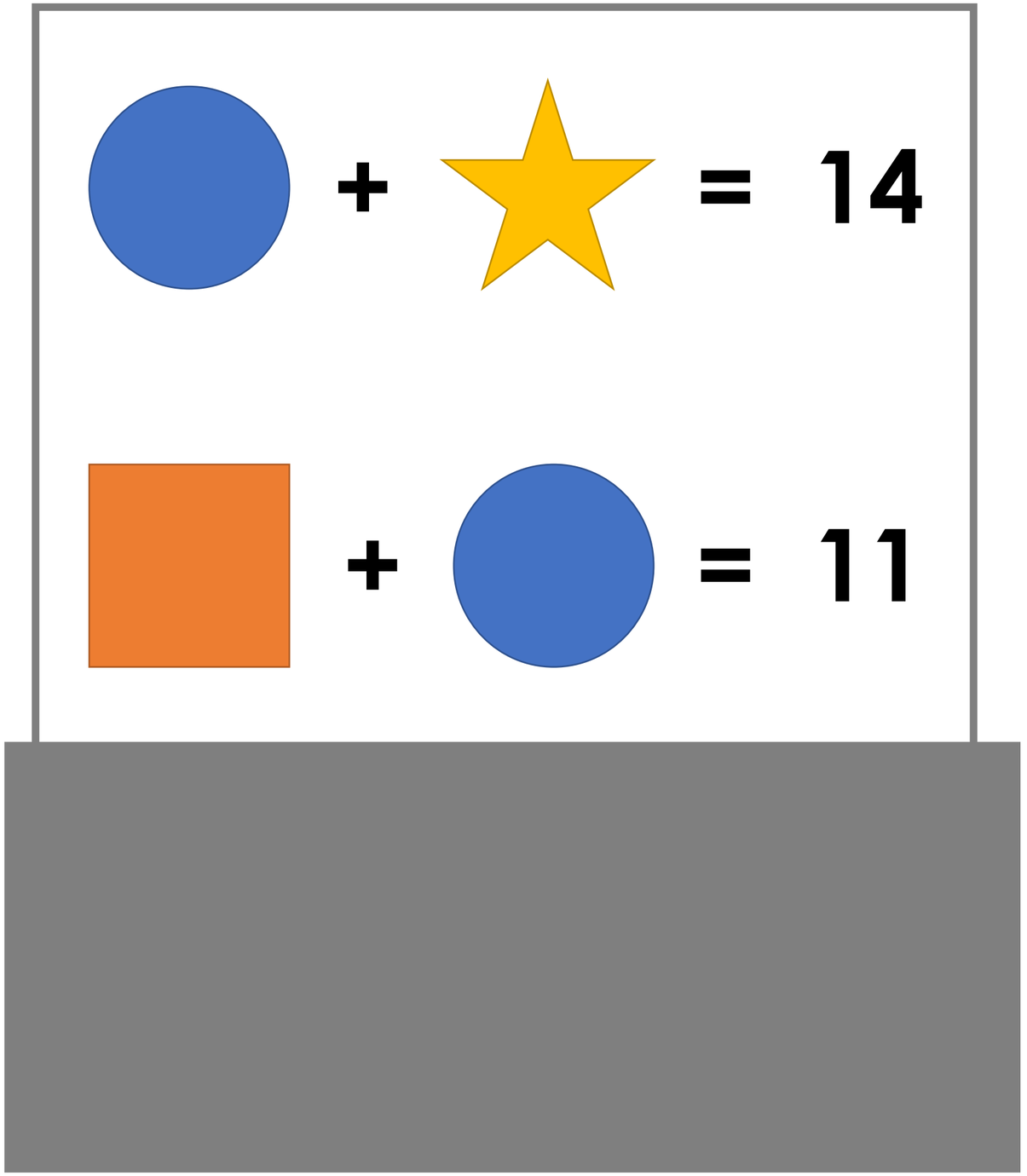
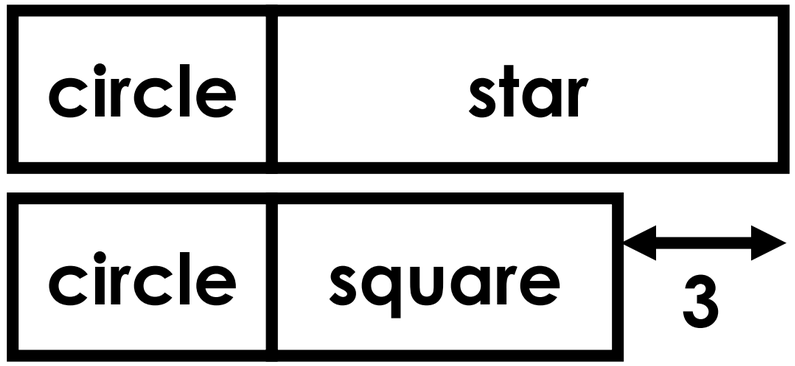
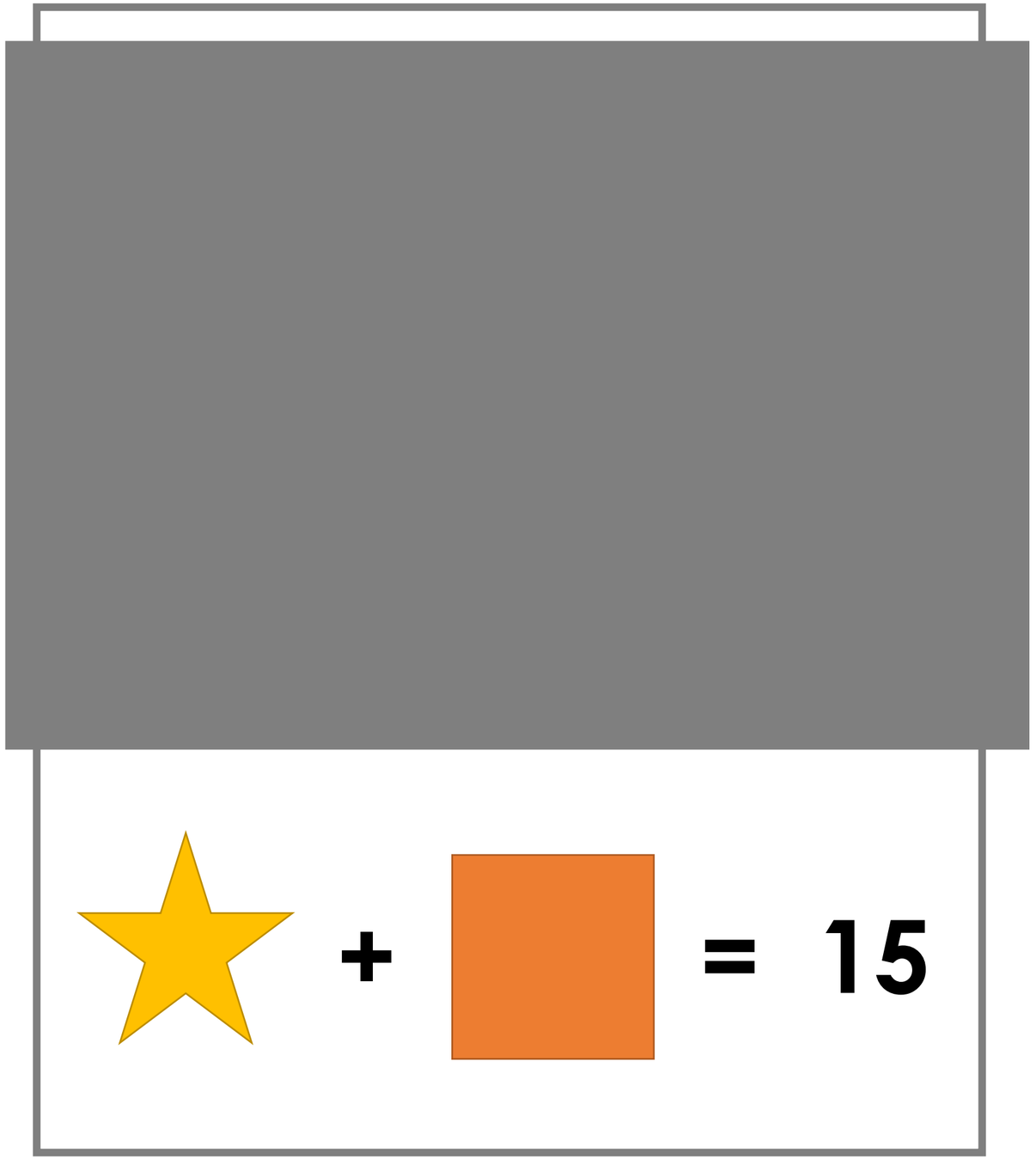


Star 3 more
than square

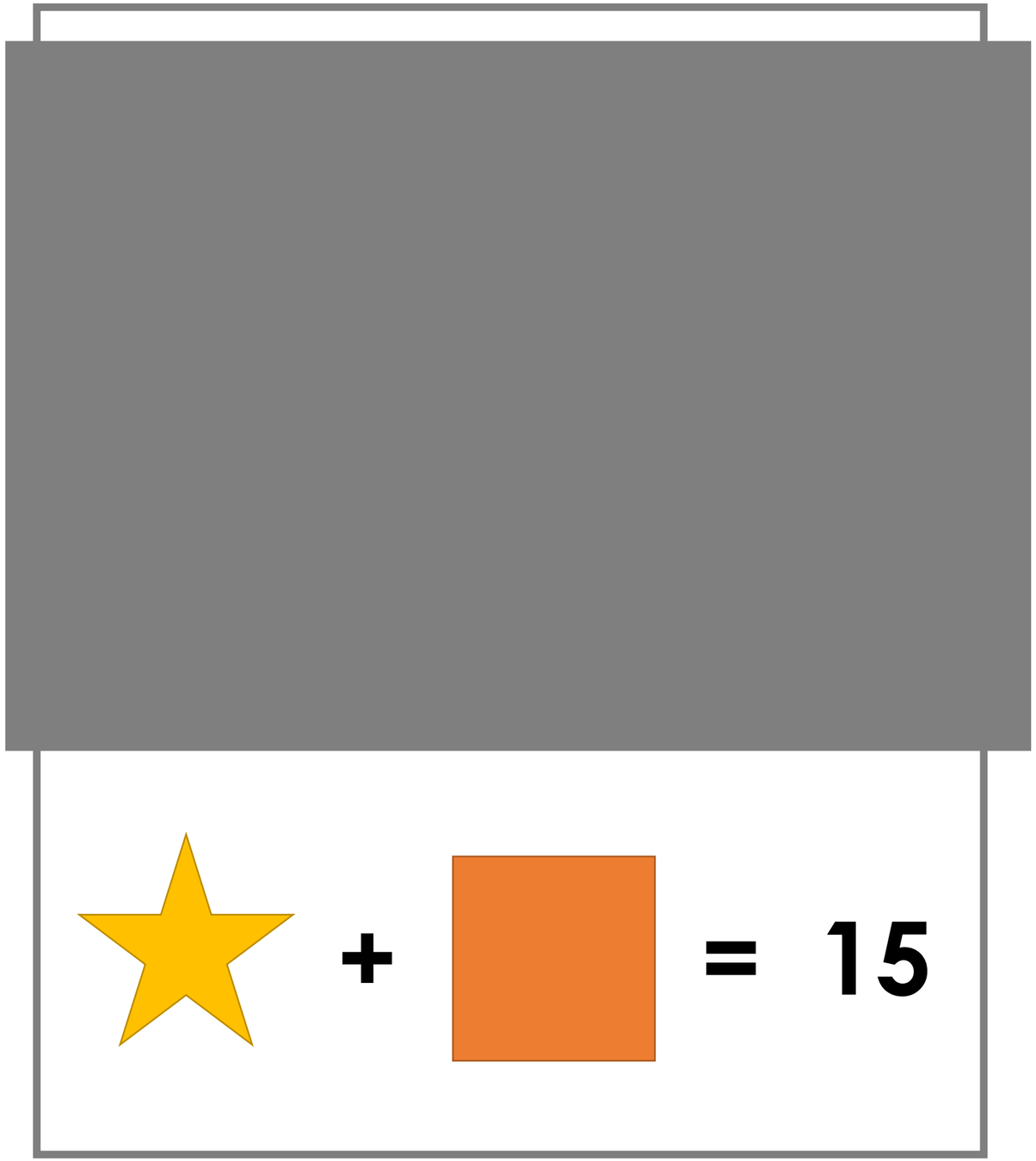


Star 3 more than square

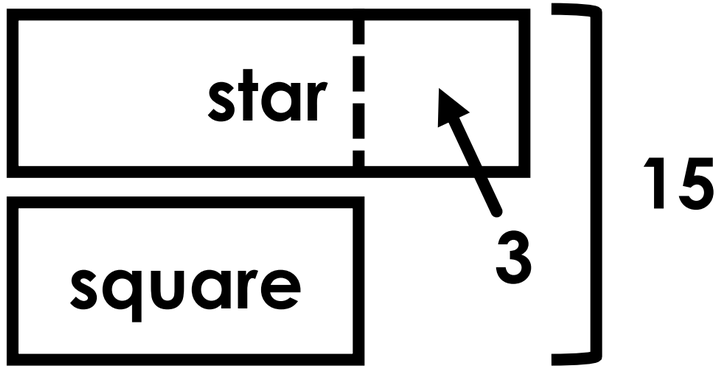


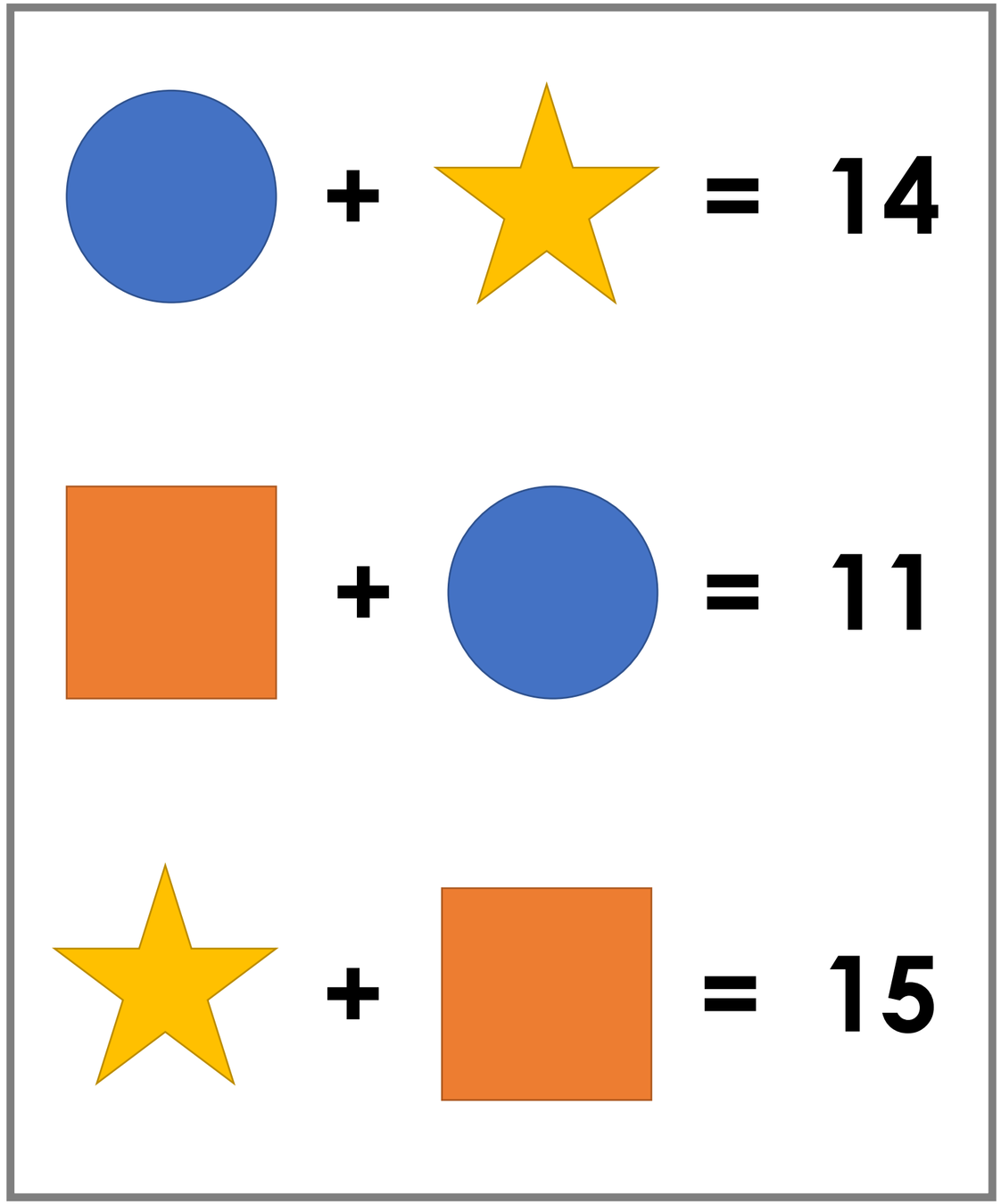


Star 3 more
than square



Star 3 more than square

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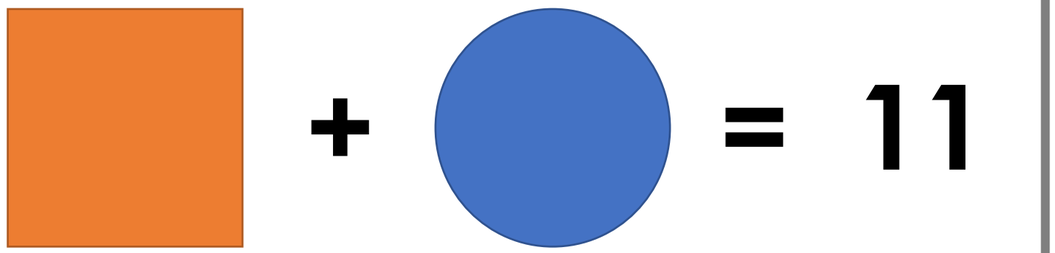
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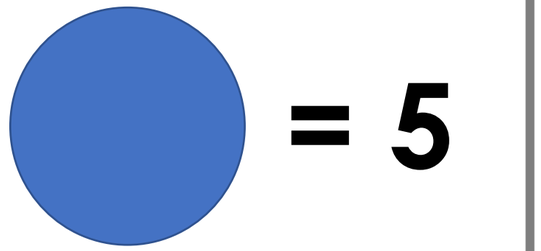
+
= 15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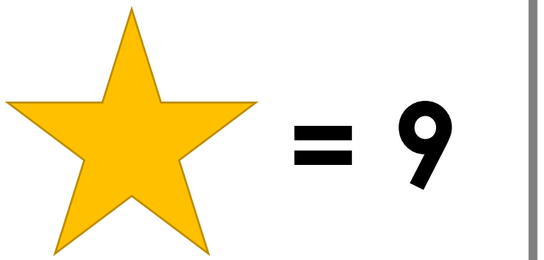
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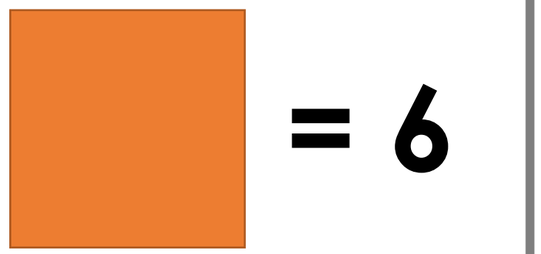

= 6

Ben and Sam weigh 90kg in total.

Jack and Ben weigh 100kg in total.

Sam and Jack weigh 80kg in total.

How much does Sam weigh?

13.2.2020

To be able to find possible of values

25.2.2020

Algebra with Mr Metcalfe

Ben and Sam weigh 90kg in total.
Jack and Ben weigh 100kg in total.
Sam and Jack weigh 80kg in total.
How much does Sam weigh?

90 = ben and sam

100 = sam and Jack

80 = Jack and Ben switch

S + J = 80kg ✓

B =

J - S = 10 : J is 10 more than S

S + ~~8~~ = 80kg ✓

↓45

J	35	10	80
S	35	↔ 10	

↑35

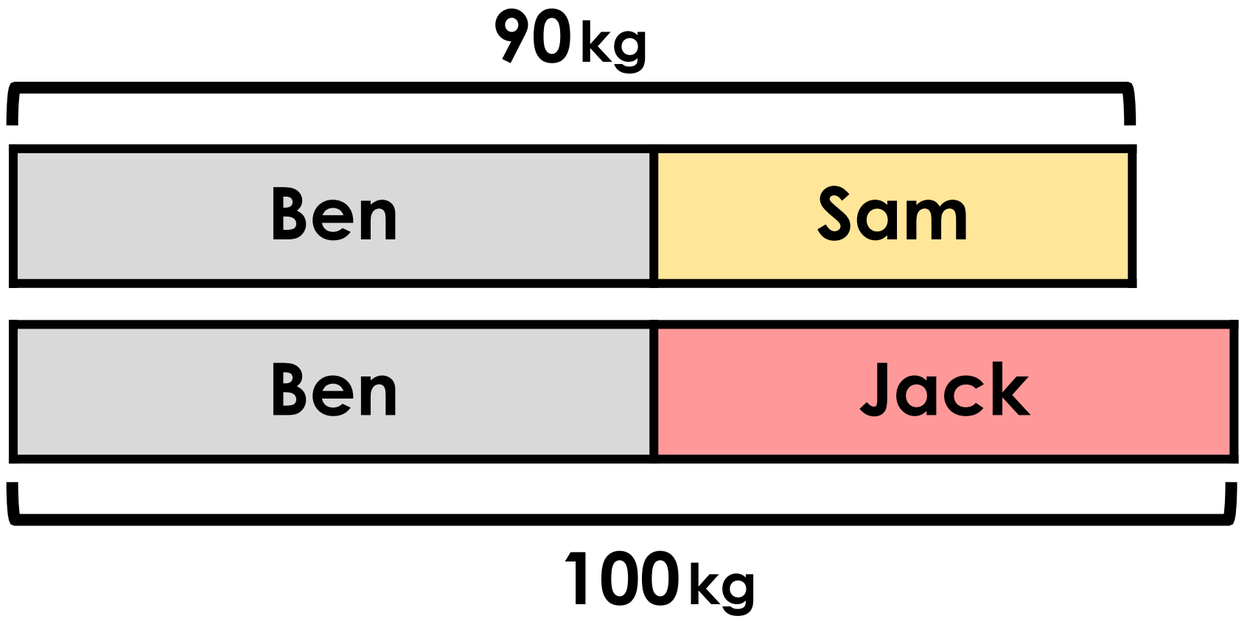
B = 55kg ✓

S = 35kg ✓ switch

J = 45kg ✓

8'10
-35
55 ✓

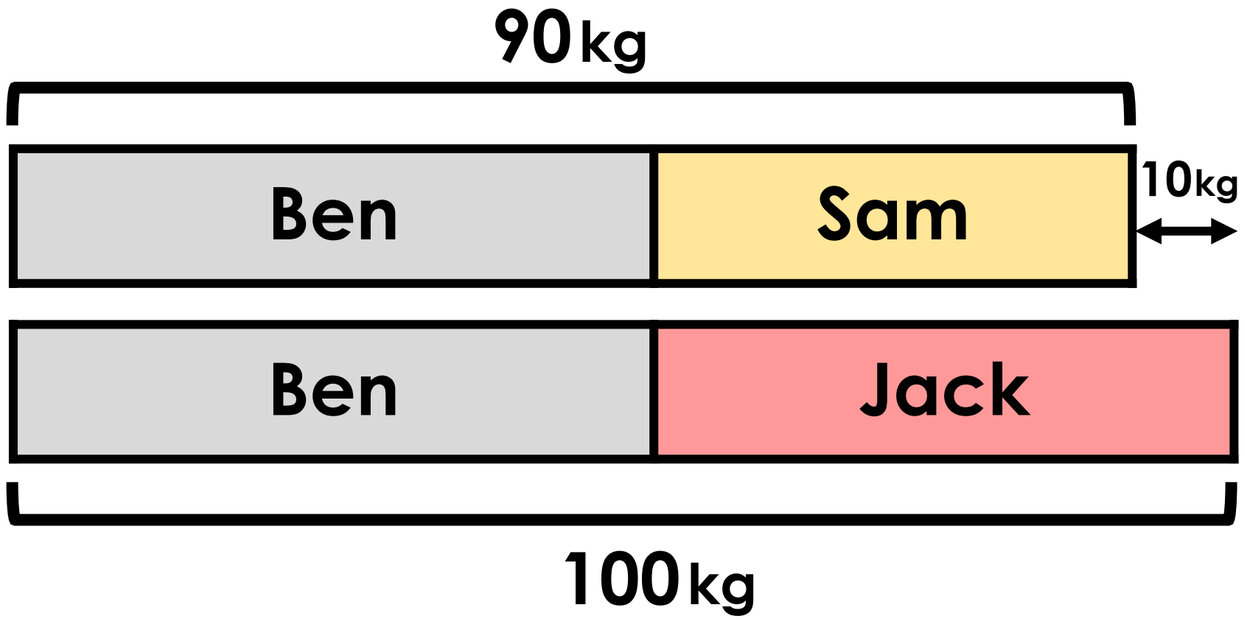
Task 31: Combined weights



Ben + Sam = 90 kg

Ben + Jack = 100 kg

Task 31: Combined weights

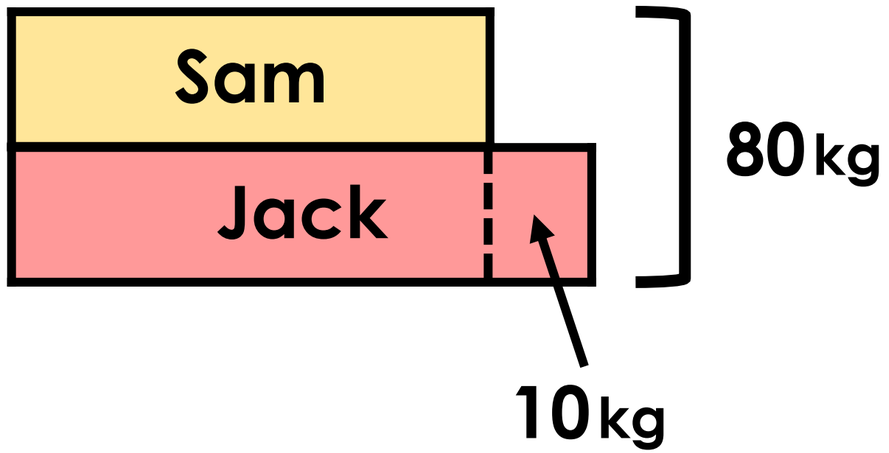


Ben + Sam = 90kg

Ben + Jack = 100kg

Jack weighs 10kg more than Sam.

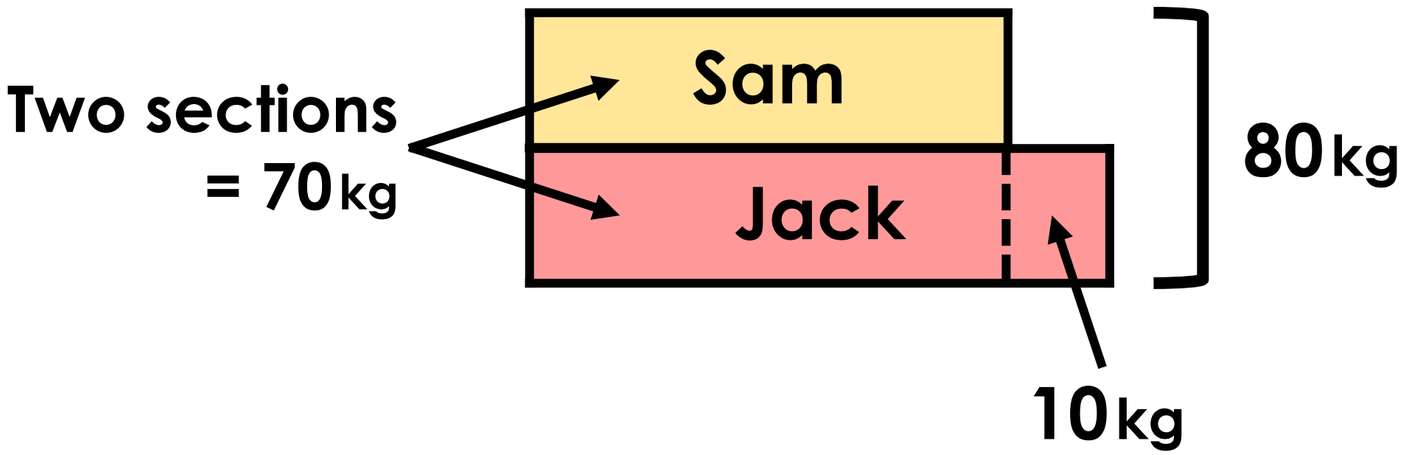
Task 31: Combined weights



Jack weighs 10kg more than Sam.

Sam and Jack weigh 80kg in total.

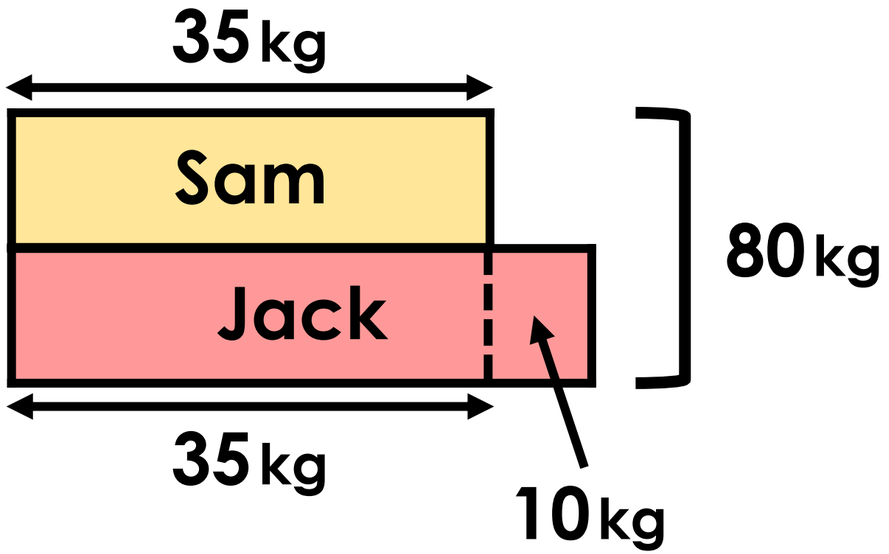
Task 31: Combined weights



Jack weighs 10kg more than Sam.

Sam and Jack weigh 80kg in total.

Task 31: Combined weights



Sam weighs 35kg

Jack weighs 10kg more than Sam.

Sam and Jack weigh 80kg in total.

EXTEND

Julie and Steph weigh 80kg in total.

Alice and Julie weigh 80kg in total.

Agree or disagree:

'Alice, Julie and Steph must all weigh 40kg.'

Steph and Alice must weigh the same.'

Julie and Steph must weigh the same.'

Julie and Steph weigh 80kg in total.

Alice and Julie weigh 80kg in total.

Agree or disagree:

1. 'Alice, Julie and Steph must all weigh 40kg.'
2. 'Steph and Alice must weigh the same.'
3. 'Julie and Steph must weigh the same.'

1. Disagree because Julie could weigh 50 KG and steph and Alice could weigh 30 KG

2. Agree because J+A=80 KG and J+S=80 KG so J and A must = the same.

3. Disagree because Julie could = 50 KG so the Steph would have to = 30 KG.

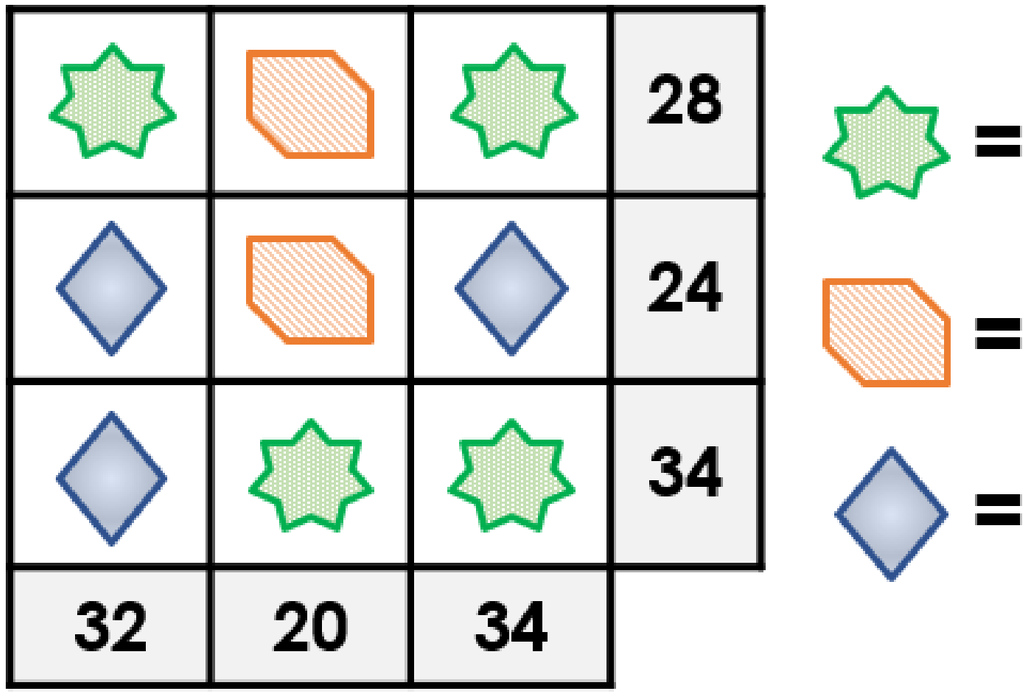
When faced with a maths problem in context, typically students focus on the scenario (its surface structure) instead of on the mathematics required to solve it (its deep structure). So even though students have been taught how to solve a particular type of word problem, when the scenario changes, students struggle to apply the solution.

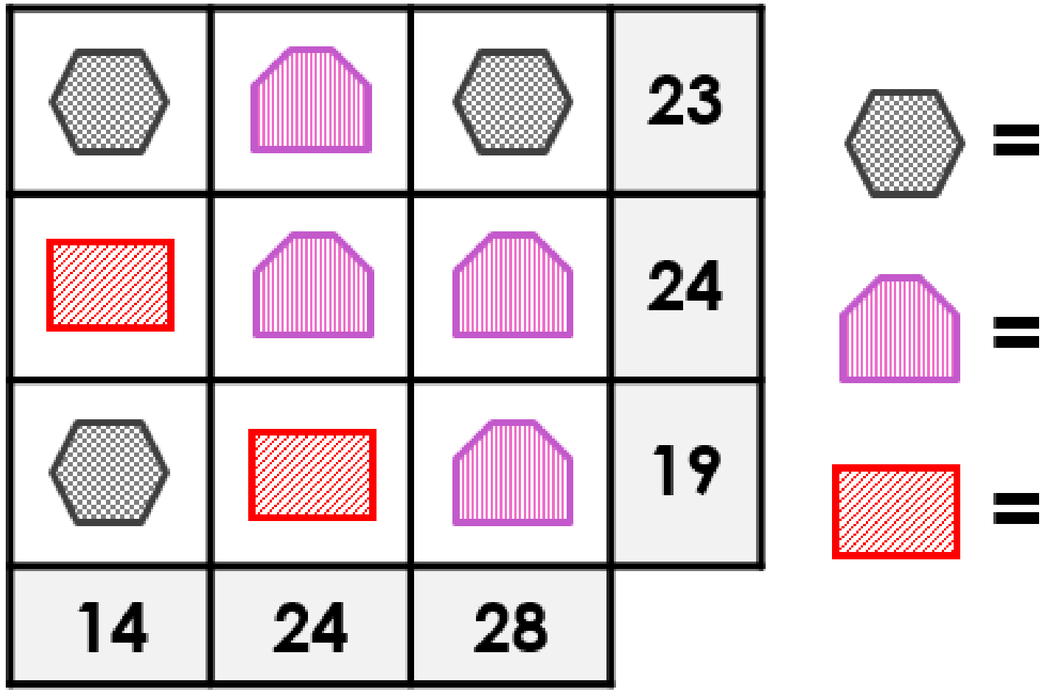
Exposure to lots of variations of problems that have the same underlying deep structure certainly helps students to look beyond the surface elements of each problem and focus on the relevant mathematics underlying it.

Craig Barton summary of **Critical thinking: why is it so hard to teach?** *Daniel Willingham*

Explore

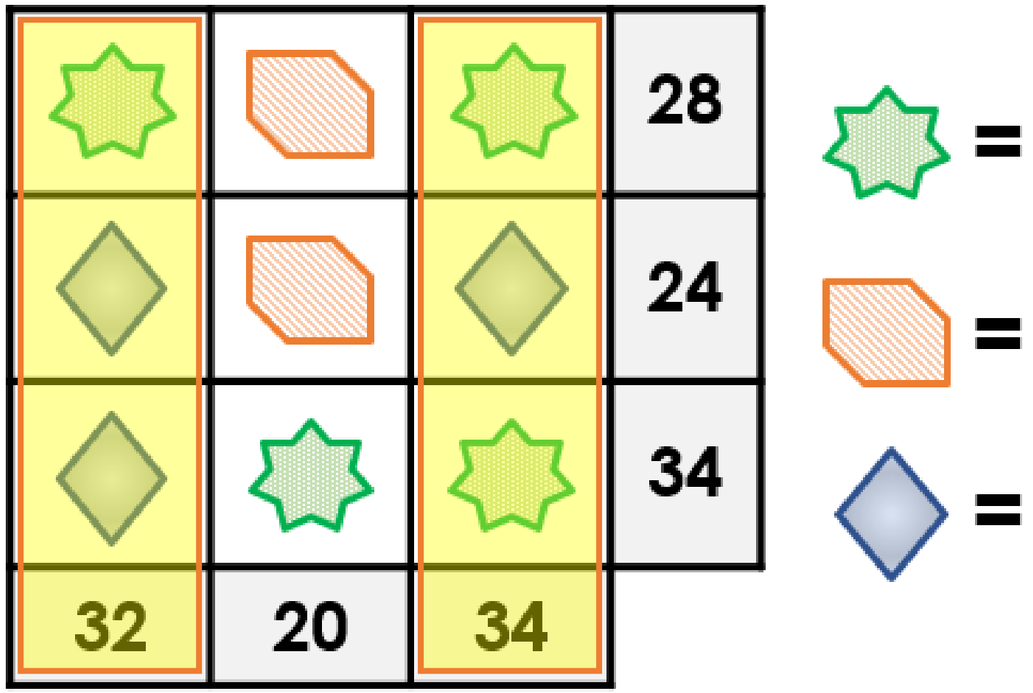
The numbers show the sum for the columns and rows.

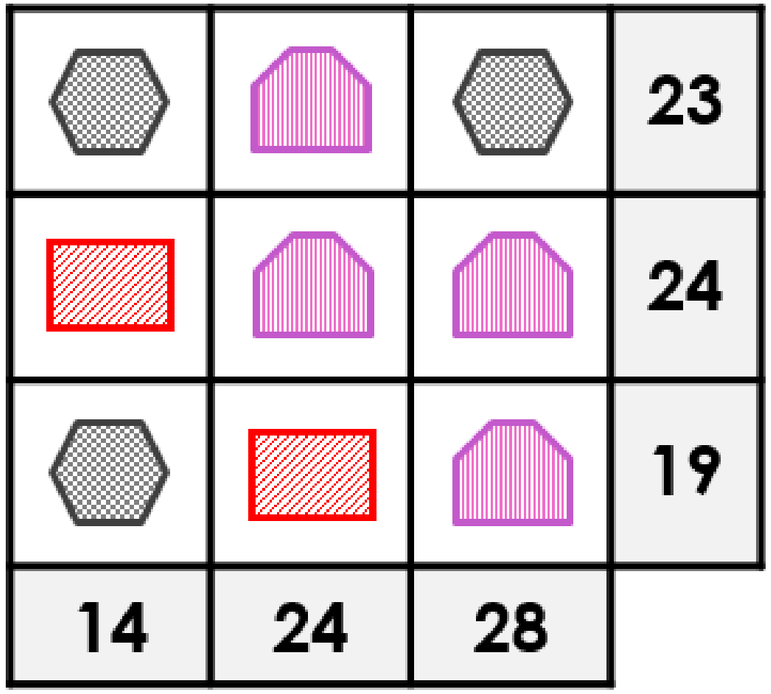


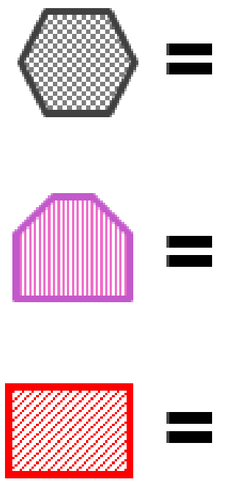


Explore

The numbers show the sum for the columns and rows.







For each number…

- How many digits
- Sum of the digits

210 **25** **308**

59 **48**

Which of these numbers are **multiples of 3?**

210 **25** **308**

59 **48**

The sum of the digits of a number is 7.

What could the number be?

The sum of the digits in a 2-digit number is 13.

The number is a multiple of 4.

What is the number?

EXPLAIN

The sum of the digits of 64 is 10.

'It's possible to keep the sum of the digits the same but change the size of the number by...'

Find different ways.

1. Add zeros e.g. 13 x 100 (add zeros) 1 x 10 = 10 (add zeros)
   1,300 x

2. Swap the numbers around eg. 43 37 73
   34 x (swapped the numbers)

3. Split the numbers up e.g. 64
   3,3,22

4. Turn into a decimal
   e.g. 0.64

EXTEND

The sum of the digits of a 3-digit number is 3.

The number is a multiple of 3.

List all the possible values for the number.

102 120 201 210 111 300

$$\frac{17}{\square} = 2\frac{\square}{\square}$$

Level 1: I can find an answer

Level 2: I can find different answers

Level 3: I know how many possible answers there are

$\frac{17}{6} = 2\frac{5}{6}$ $\frac{17}{7} = 2\frac{3}{7}$ $\frac{17}{8} = 2\frac{1}{8}$

I know there is only 3 ways because if you did give the whole number will become 3 and if you did nine and the whole number will change to 1.

$\frac{17}{5} = 3\frac{2}{5}$ $\frac{17}{9} = 1\frac{8}{9}$

EXPLAIN

Circle the fractions that are **more than 3 and less than 4.**

$\frac{14}{3}$ $\frac{14}{4}$ $\frac{21}{7}$ $\frac{11}{3}$ $\frac{19}{5}$

Explain how you know.

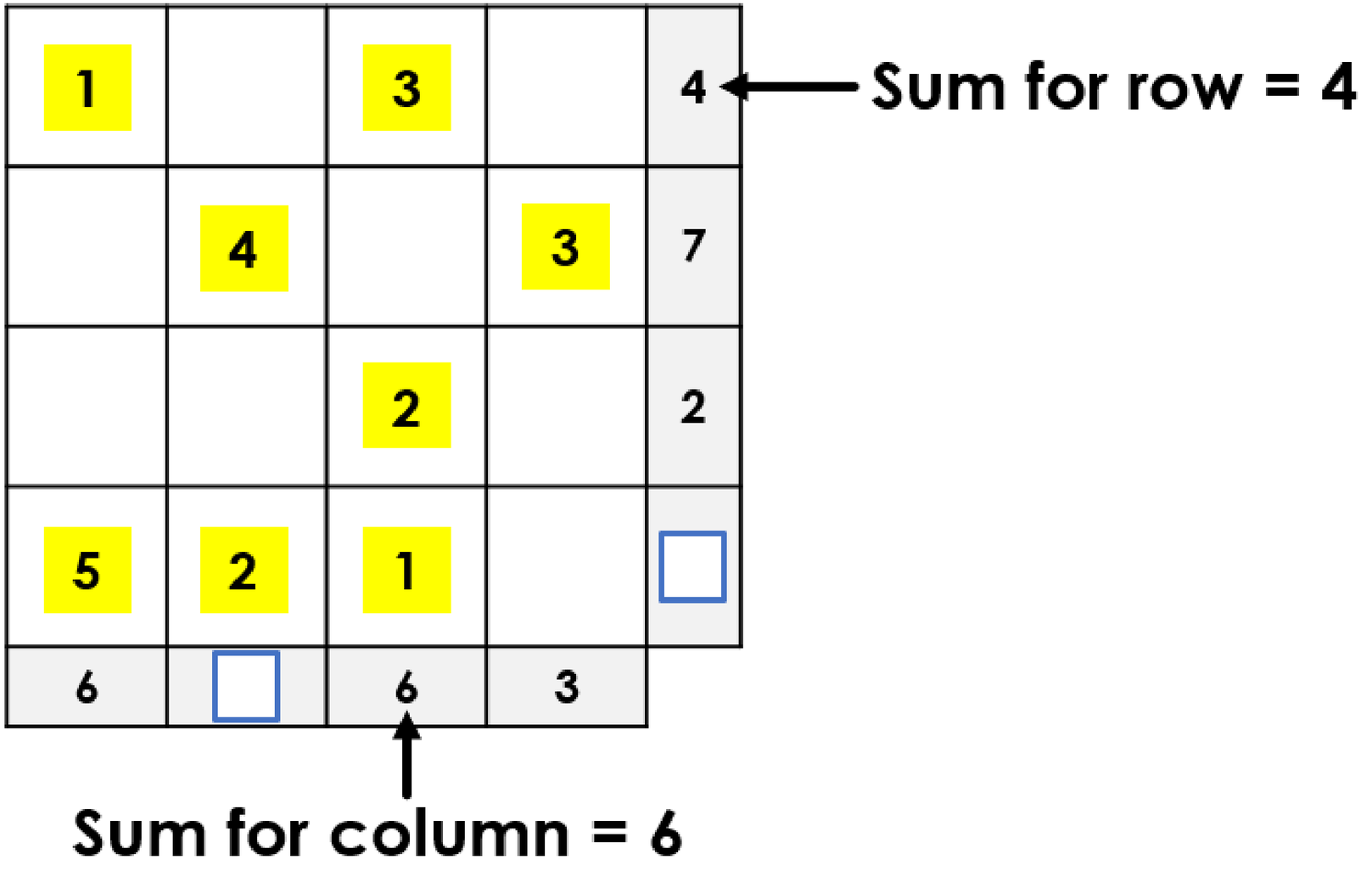
4 × 3 = 12 but we have 2 left over so it's $3\frac{2}{4}$
3 × 3 = 9 but we have 2 left over so it is $3\frac{2}{3}$
5 × 3 = 15 but we have 4 left over so it's $3\frac{4}{5}$

EXTEND

Fill in the missing numerators so the fractions are written ***in order of size****, from smallest to largest:*

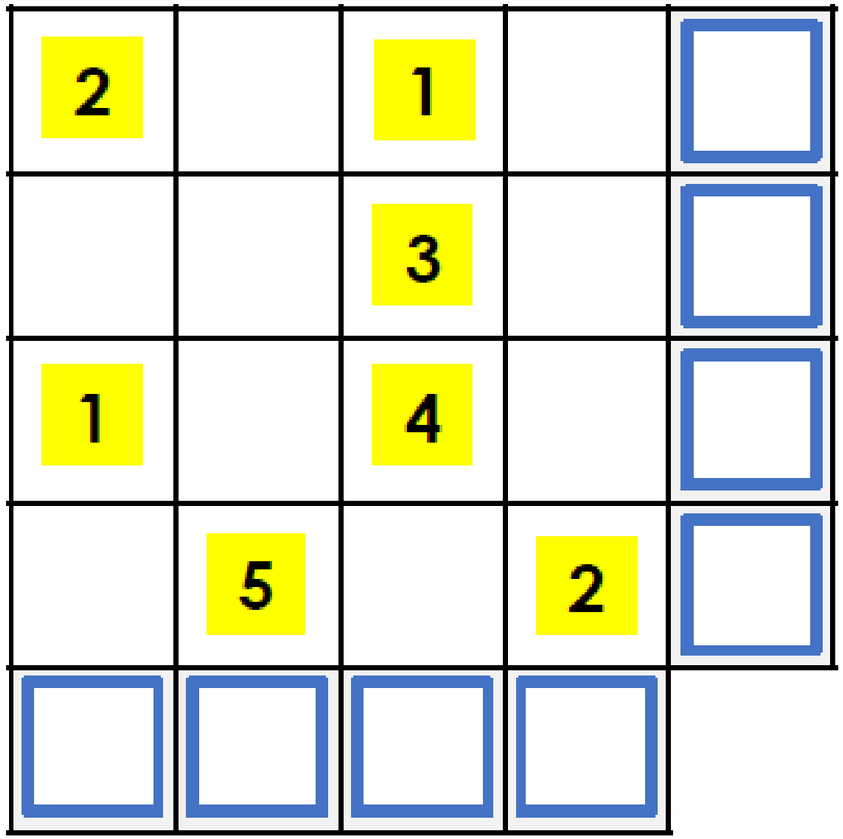
$4\frac{1}{5}$ $\frac{21}{5}$ $\frac{27}{6}$ $4\frac{3}{6}$ $\frac{32}{7}$ $4\frac{4}{7}$ $\frac{39}{8}$ $4\frac{7}{8}$

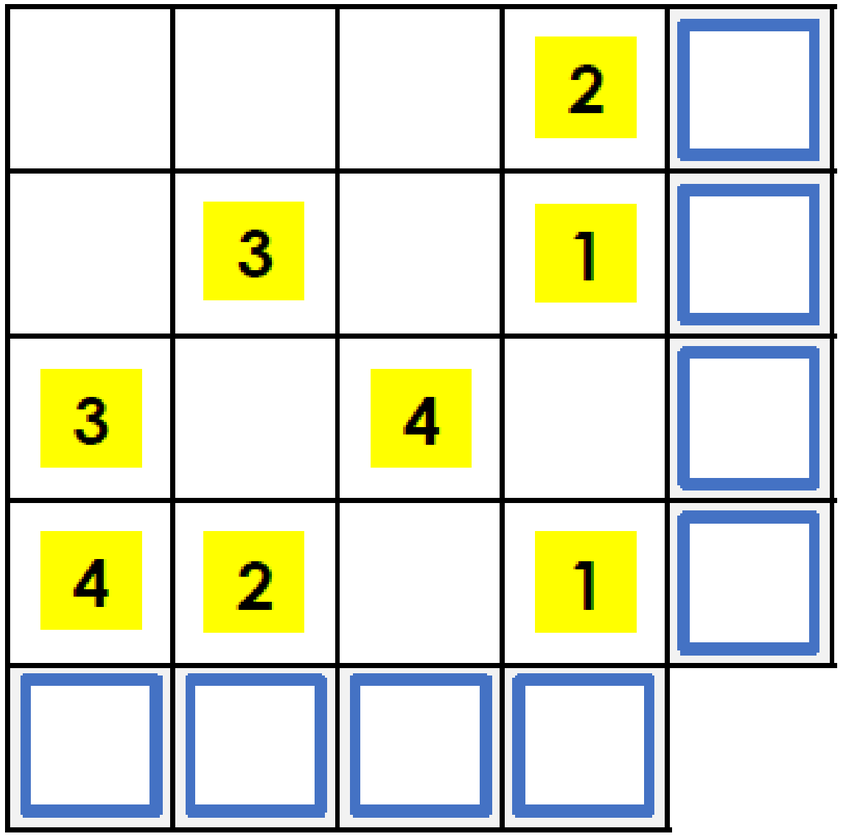
smallest ***largest***


1
3
4
Sum for row = 4
4
3
7
2
2
5
2
1
6
6
3
Sum for column = 6

Find the sum of the numbers in each column and row.

Fill the gaps.





Task 7: Digits in a Grid B

Find the sum of the numbers in each column and row.

Correct or incorrect? ✓ **or** ✗

2	4			6
		3		3
		1	7	8
6	5			9
8	8	4	7	

	3	2		5
	3		5	8
	1	4		5
4	2			6
4	9	6	5	

Put the numbers 1, 2, 3, 4, 5, 6 on the grid so the sums for the columns and rows are correct:

				6
				6
				8
				1
9	4	5	3	

1	2	3
4	5	6

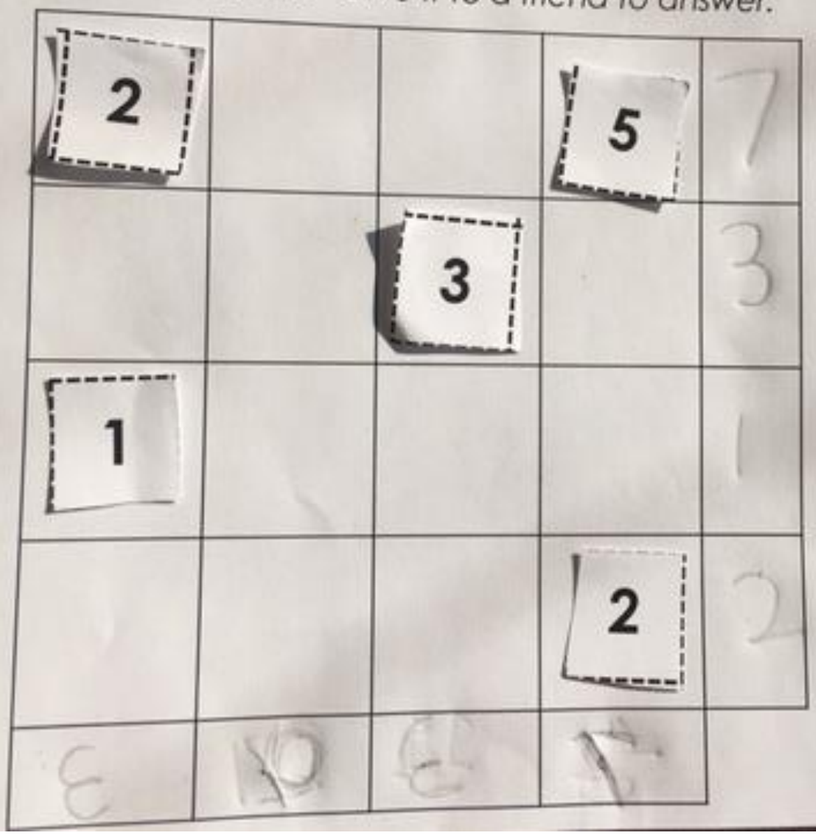
Draw the answer:

				6
				6
				8
				1
9	4	5	3	

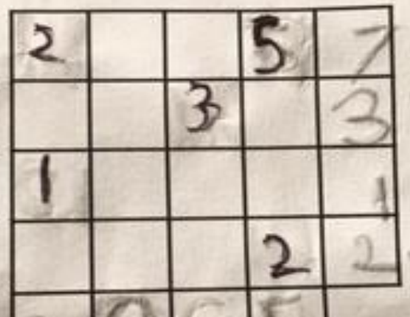
Task 7: Digits in a Grid

D

Design your own puzzle. Write in the sums for the columns and rows. *Give it to a friend to answer.*



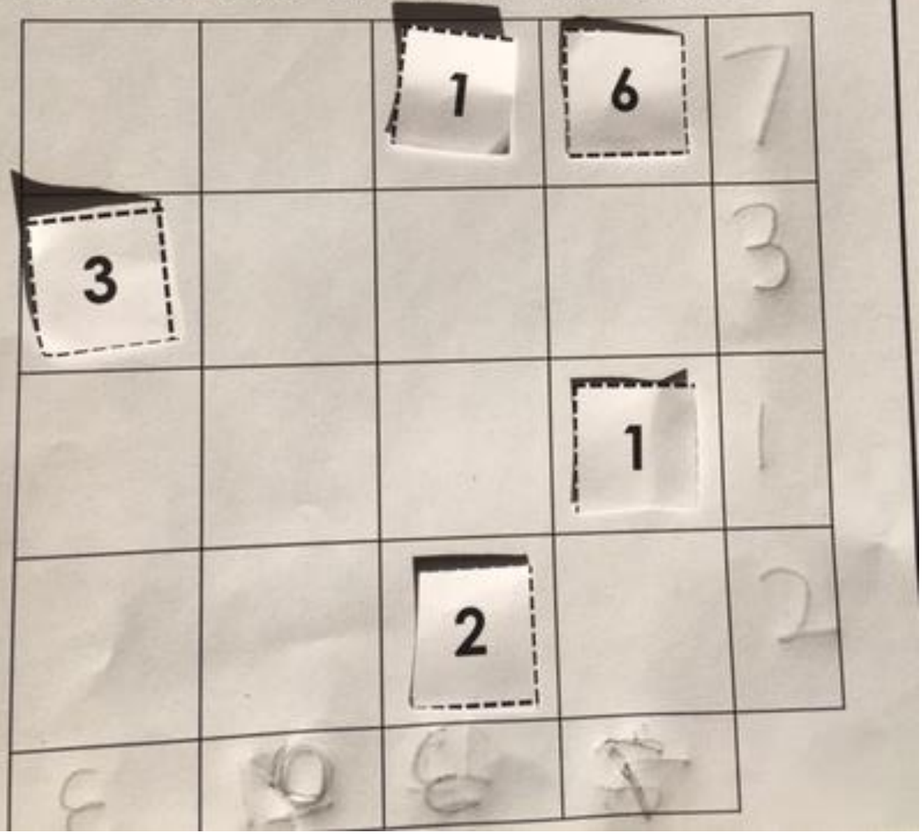
Answer Grid:



Task 7: Digits in a Grid

D

Design your own puzzle. Write in the sums for the columns and rows. *Give it to a friend to answer.*



Extend:

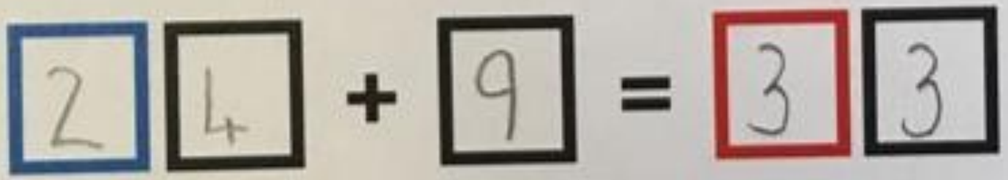
Put in **two more numbers** to make each grid correct:

4			5	9
	2			7
			3	3
		6		6
9	2	6	8	

4		3		7
			6	12
		1		1
	5			7
6	5	10	6	

BUILD

Fill the gaps. *Answer in 3 different ways.*



For each number sentence, look at the digit in the blue box and the digit in the red box. **What do you notice?**

Each time, the digit in the red box is… *more/less*

TASK

Use the digits to complete the number sentence.

You can only use each digit once in each number sentence.

□□ + □ = □□

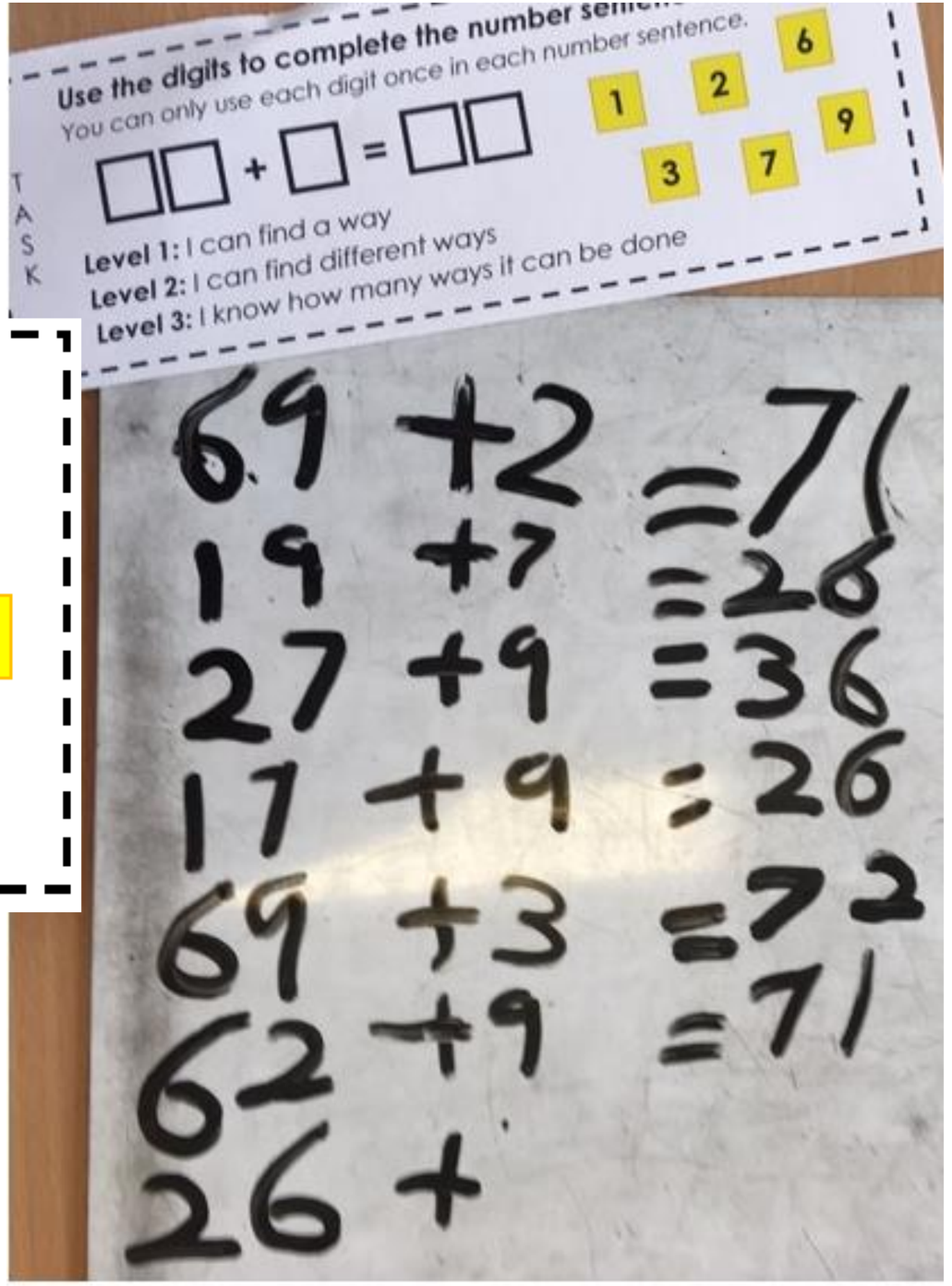
1 2 6 3 7 9

Level 1: *I can find a way*

Level 2: *I can find different ways*

Level 3: *I know how many ways it can be done*

Example: *16 + 3 = 19 is* ***not an answer*** *because it uses 1 twice.*



Choose 3 consecutive numbers.

Choose 3 consecutive numbers.

Add them.

Choose 3 consecutive numbers.

Add them.

Multiply the middle number by 3.

What do you notice?

TASK

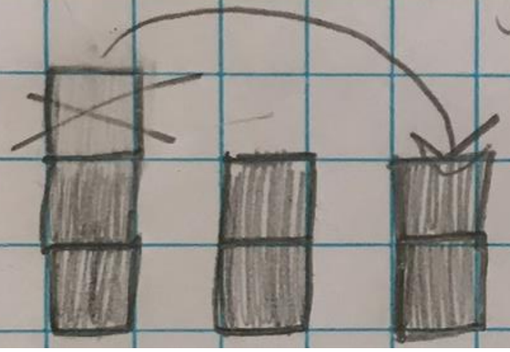
Think of **three consecutive numbers**. These are your numbers.

Add your numbers.

Multiply your middle number by 3.

What do you notice? Explain.

1, 2, 3 is the same as 2 X 3 because for 1 theres one less and 3 theres one more so if your 3 and you give 1, 1 of your numbers it will be three twos. So its the same just shown differently. Example:



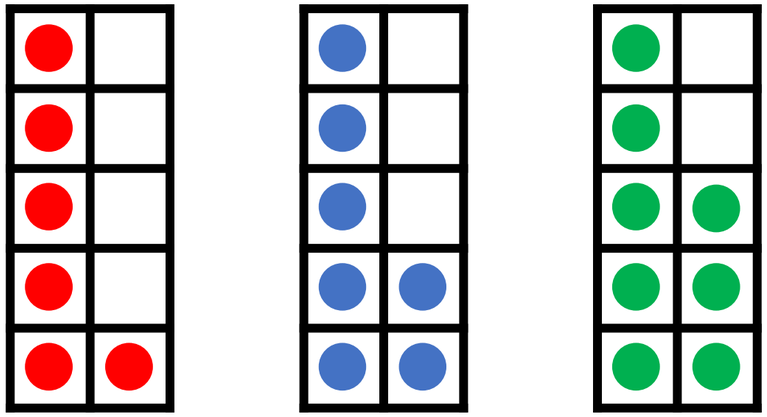
All two.

Task 11: Consecutive numbers

Three consecutive numbers: **6, 7, 8**

Task 11: Consecutive numbers

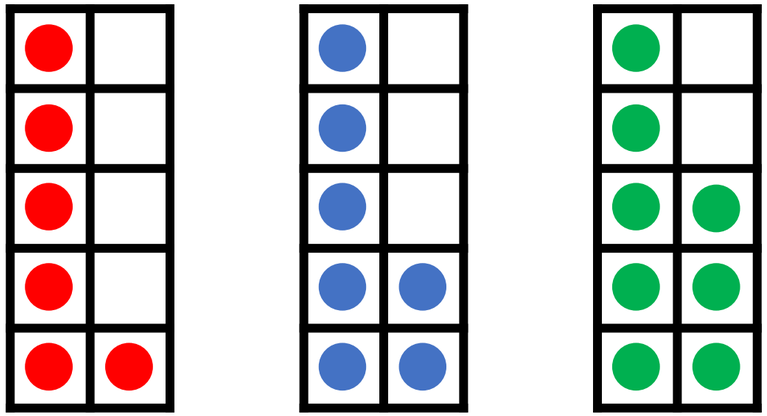
Three consecutive numbers: **6, 7, 8**



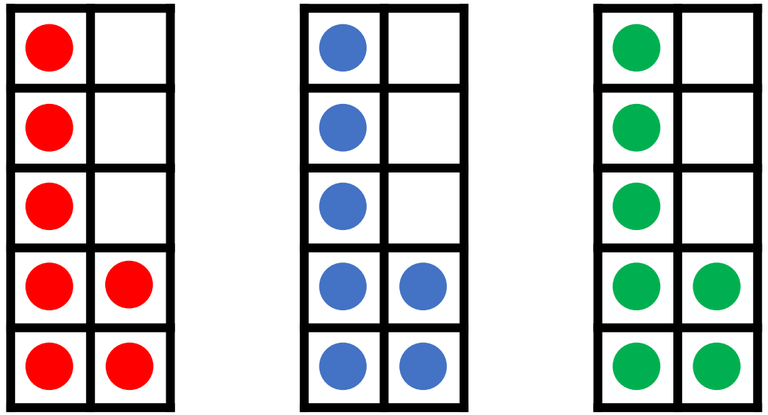
6 + 7 + 8 = 21

Task 11: Consecutive numbers

Three consecutive numbers: **6, 7, 8**



6 + 7 + 8 = 21



3 × 7 = 21

EXTEND

These consecutive numbers add up to make 18:

5, 6 and 7 3, 4, 5 and 6

Which consecutive numbers add up to make 30?

There are 3 possible answers.

1) 9, 10, 11

2) 6, 7, 8, 9

3) 4, 5, 6, 7, 8

John goes on a 3-day holiday.

He makes 15 sandcastles.

Each day he makes one more sandcastle than the day before.

How many sandcastles does he make each day?

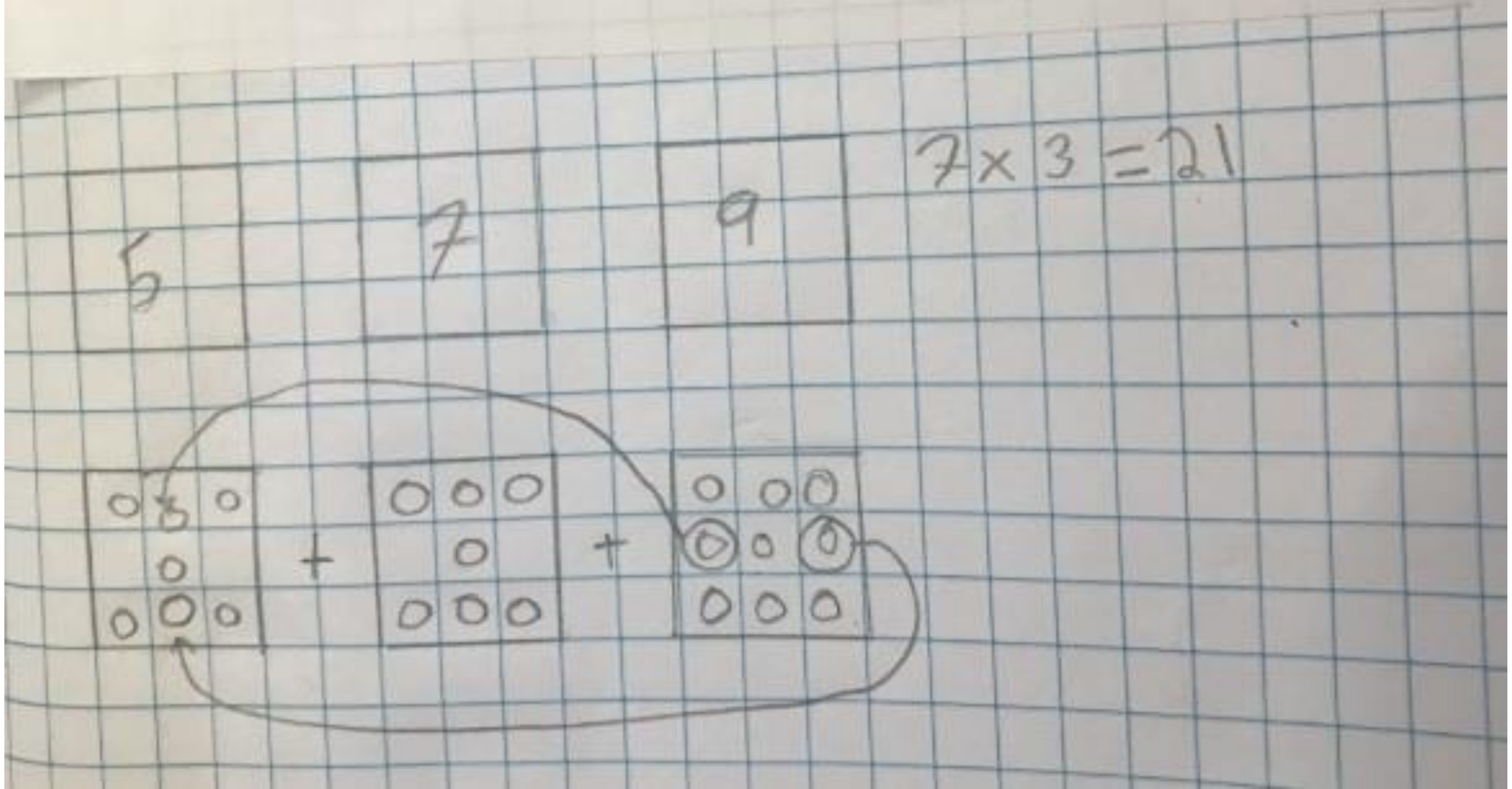
Challenge 1

Sarah goes on a 3-day holiday.

She makes 21 sandcastles.

Each day she makes two more sandcastles than the day before.

How many sandcastles does she make each day?



Challenge 3

Zack goes on a 5-day holiday.

He makes 80 sandcastles.

Each day he makes four sandcastles fewer than the day before.

How many sandcastles does he make each day?

24 20 16 12 8

80 ÷ 5 = 16

A consistent approach to problem-solving

Managing cognitive load, building pre-skills

Planning PS in sequences of lessons *Use of related tasks*

Showing deep structures, modelling expert thinking

Representations *Balance of teaching and exploration*

Finding and developing problem-solving tasks

Developing task families *Differentiation, challenge*

Consistency within a school

Creating and sharing a shared approach

Small Steps

To calculate this digit, I must work out $\square + 3 = 8$

$$\begin{array}{r} \square 29 \\ +\ 34\square \\ \hline 874 \end{array}$$

To calculate this digit, I must work out $9 + \square = 14$

Extend

$$\begin{array}{r} 2\square 1 \\ +\ \ 8\square \\ \hline 316 \end{array} \qquad \begin{array}{r} 4\square 0 \\ +\ 22\square \\ \hline 717 \end{array} \qquad \begin{array}{r} 253\square \\ +\ \square 32 \\ \hline 3169 \end{array}$$

$$\begin{array}{r} 4\square 5 \\ +\ 21\square \\ \hline 649 \end{array} \qquad \begin{array}{r} 4\square 9 \\ +\ 21\square \\ \hline 645 \end{array} \qquad \begin{array}{r} 2\square 43 \\ +\ \ 6\square 7 \\ \hline 3060 \end{array}$$

How Many Ways?

Complete the calculation:

□57 + 34□ = 1□02

Level 1: *I can find an answer*

Level 2: *I can find different answers*

Level 3: *I know how many answers there are*

Explain

Which question can be answered in **one way?**

Which question can be answered in **different ways?**

$$\begin{array}{r} \square 2 \square \\ +\ 1 \square 4 \\ \hline 1 \square 2\,7 \end{array} \qquad \begin{array}{r} \square 2 \square \\ +\ 3 \square 4 \\ \hline 1 \square 2\,7 \end{array}$$

Explain why.

How Many Ways?

Complete the calculation:

6	5	7	+	3	4	5	=	1	0	0	2

Level 1: *I can find an answer*

Level 2: *I can find different answers*

Level 3: *I know how many answers there are*

Explain

Which question can be answered in **one way?**

Which question can be answered in **different ways?**

$$\begin{array}{r} \square 2 \square \\ +\ 1 \square 4 \\ \hline 1 \square 2\,7 \\ \hline \end{array} \qquad \begin{array}{r} \square 2 \square \\ +\ 3 \square 4 \\ \hline 1 \square 2\,7 \\ \hline \end{array}$$

Explain why.

How Many Ways?

Complete the calculation:

9	5	7	+	3	4	5	=	1	3	0	2

Level 1: *I can find an answer*

Level 2: *I can find different answers*

Level 3: *I know how many answers there are*

Explain

Which question can be answered in **one way?**

Which question can be answered in **different ways?**

$$\begin{array}{r} \square 2 \square \\ +\ 1 \square 4 \\ \hline 1 \square 2\,7 \\ \hline \end{array} \qquad \begin{array}{r} \square 2 \square \\ +\ 3 \square 4 \\ \hline 1 \square 2\,7 \\ \hline \end{array}$$

Explain why.

How Many Ways?

Complete the calculation:

9	5	7	+	3	4	5	=	1	3	0	2

Level 1: *I can find an answer*

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Explain

Which question can be answered in **one way?**

Which question can be answered in **different ways?**

Explain why.

How Many Ways?

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Level 1: *I can find an answer*

Level 2: *I can find different answers*

Level 3: *I know how many answers there are*

Explain

Which question can be answered in **one way?**

Which question can be answered in **different ways?**

		9	2	3				9	2	3	
+	1	0	4		+	3	0	4			
1	0	2	7		1	2	2	7			

Explain why.

How Many Ways?

Complete the calculation:

9	5	7	+	3	4	5	=	1	3	0	2

Level 1: *I can find an answer*

Level 2: *I can find different answers*

Level 3: *I know how many answers there are*

Explain

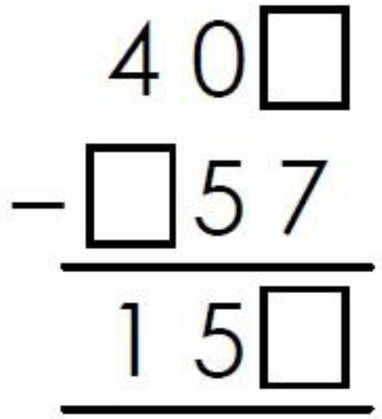
Which question can be answered in **one way?**

Which question can be answered in **different ways?**

		9	2	3				8	2	3	
+	1	0	4		+	3	0	4			
1	0	2	7		1	1	2	7			

Explain why.

How Many Ways?



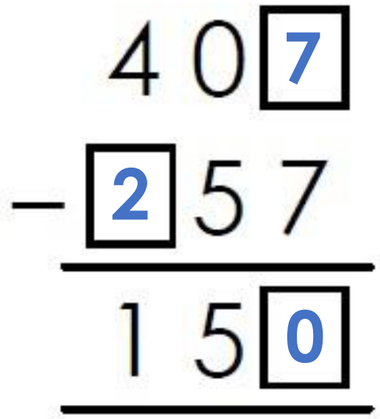
Level 1: *I can find an answer.*

Level 2: *I can find different answers.*

Level 3: *I know how many answers there are.*

Extend: *design your own missing digits subtraction question that can be answered in different ways.*

How Many Ways?



Level 1: *I can find an answer.*

Level 2: *I can find different answers.*

Level 3: *I know how many answers there are.*

Extend: *design your own missing digits subtraction question that can be answered in different ways.*

Zara

Sandwich: £1.80
Drink: 30p

Zara

Sandwich: £1.80
Drink: 30p

How many sandwiches can she afford?

Zara

Sandwich: £1.80
Drink: 30p

How much change does she get?

Sports Direct

Tennis balls: £1.50 each

JD Sports

Tennis balls: £5 for four

One calculation

Multi-step calculation

Kelsey buys some fairy cakes for a party.

She buys 38 packs.

There are 6 fairy cakes in each pack.

189 fairy cakes are eaten.

How many fairy cakes does Kelsey have left?

Kelsey buys some fairy cakes for a party.

She buys ☐ packs.

☐ fairy cakes are eaten.

How many fairy cakes does Kelsey have left?

☐ packs of cakes.

☐ cakes per pack.

☐ cakes eaten at the party.

☐ cakes left.

10 packs of cakes.

6 cakes per pack.

47 cakes eaten at the party.

☐ cakes left.

☐ packs of cakes.

4 cakes per pack.

26 cakes eaten at the party.

2 cakes left.

Recap

Build the pre-requisite skills

Teach structures, uncover key aspects of thinking

Tasks with different entry points e.g. ‘How Many Ways?’

Show connections between tasks with similar deep structures or different surface features

Adapt to your context