Embedding Reasoning Routines

Making mathematical reasoning a habit for all children. Teach reasoning skills, use consistent structures, facilitate reflection on learning and enable creativity.

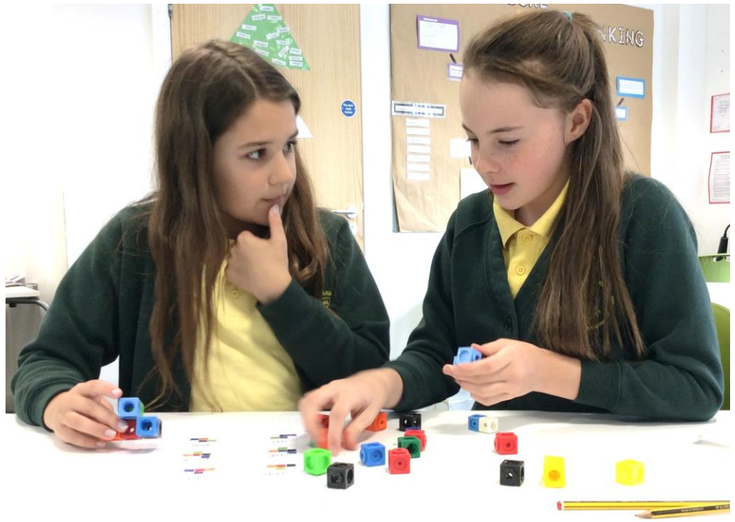
www.iseemaths.com **Y1 – Y6**

Make logical leaps

Understand concepts

Creative

Flexible

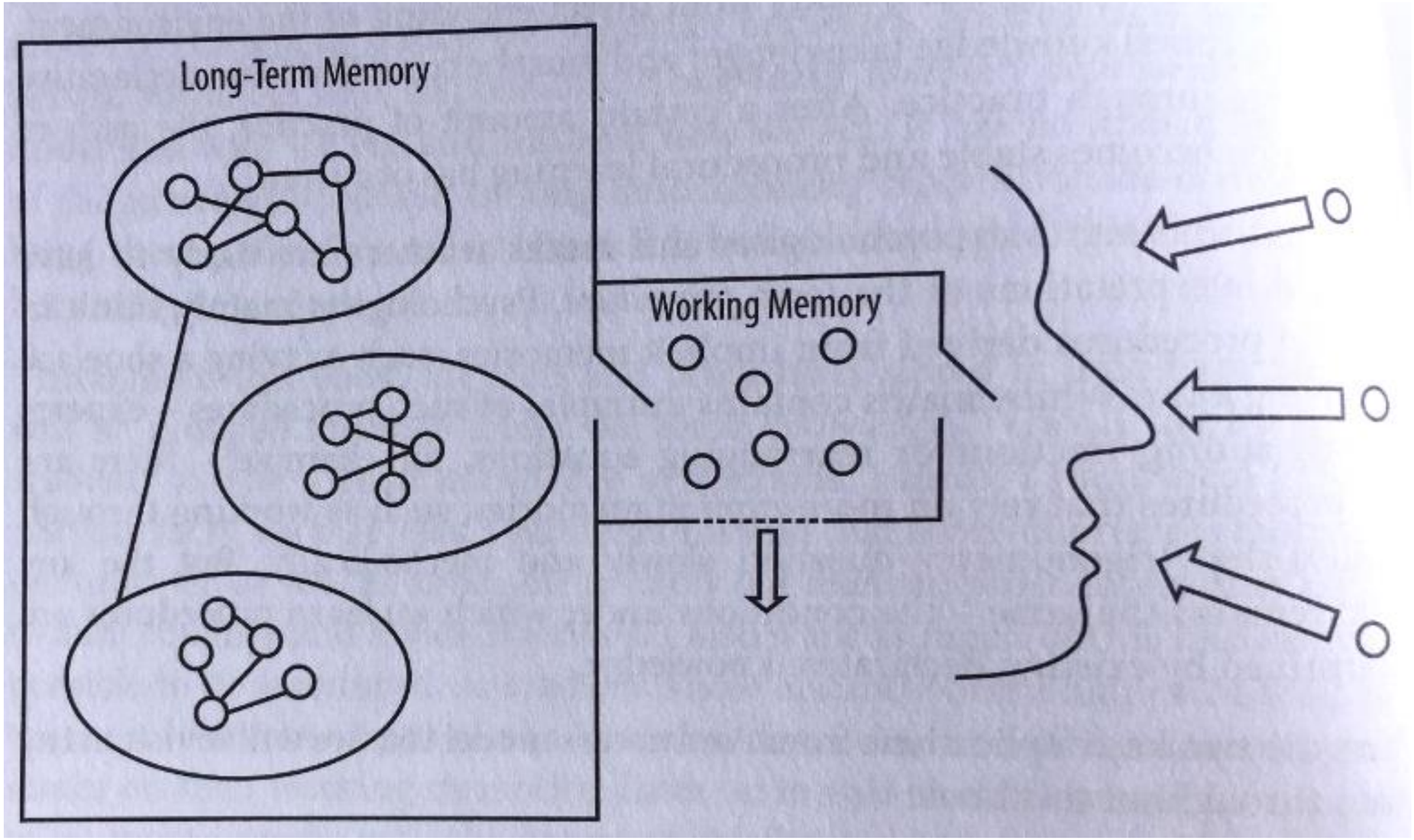


Reflective

Ordered

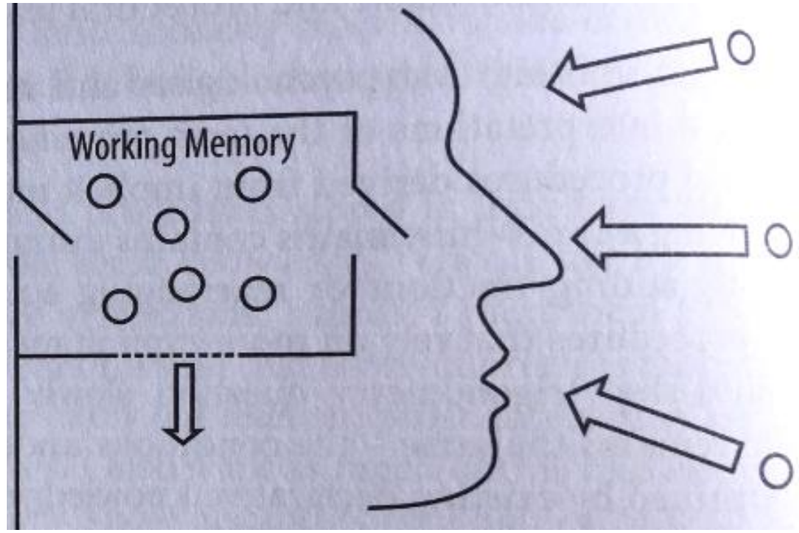
See relationships, generalise

Show understanding in different ways



Novice → Expert

Creating habits to protect our limited attention:



'Show me your whiteboard'

Triggers for silence

'Eyes on me' Vs 'Track the speaker'

Discussions: inclusive, deep, managing thinking

Fairness and motivation

Use of equipment

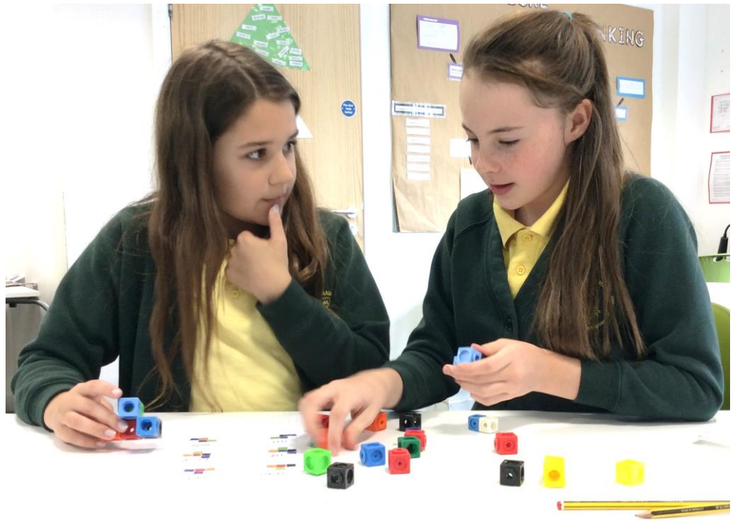
Systems for recording

Discussion and silence

Make logical leaps
I know… so…

Understand concepts
Explain the Mistakes

Creative
Design questions



Flexible
Different Methods
Rank by Difficulty

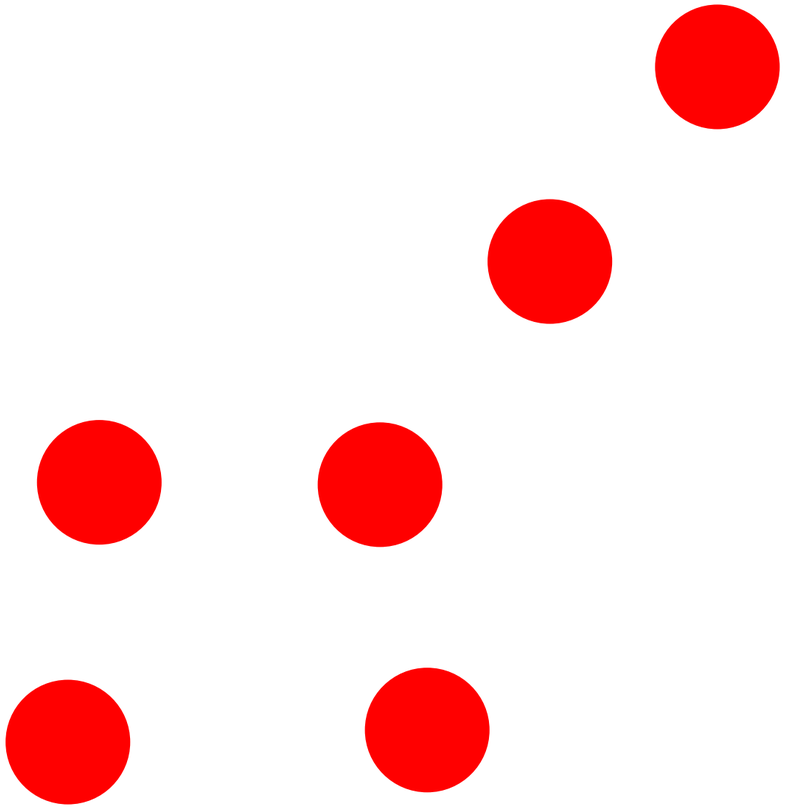
Reflective
Explicit learning behaviours

Ordered
How Many Ways?
Different Ways

See relationships, generalise
Small Difference Questions

Show understanding in different ways
Use of representations

Explaining Other Perspectives



Understand concepts

Explain the Mistakes

Explain the Mistakes

42 × 6

Mistake A:

	40	2
6	24	12

$$\begin{array}{r} 24 \\ +\ 12 \\ \hline 36 \end{array}$$

Mistake B:

	40	2
6	240	12

$$\begin{array}{r} 240 \\ +\ 12 \\ \hline 360 \end{array}$$

Mistake C:

	40	2
6	240	12

$$\begin{array}{r} 240 \\ +\ 12 \\ 6 \\ \hline 258 \end{array}$$

Answer the question correctly.

Explain the mistake

5 + 3

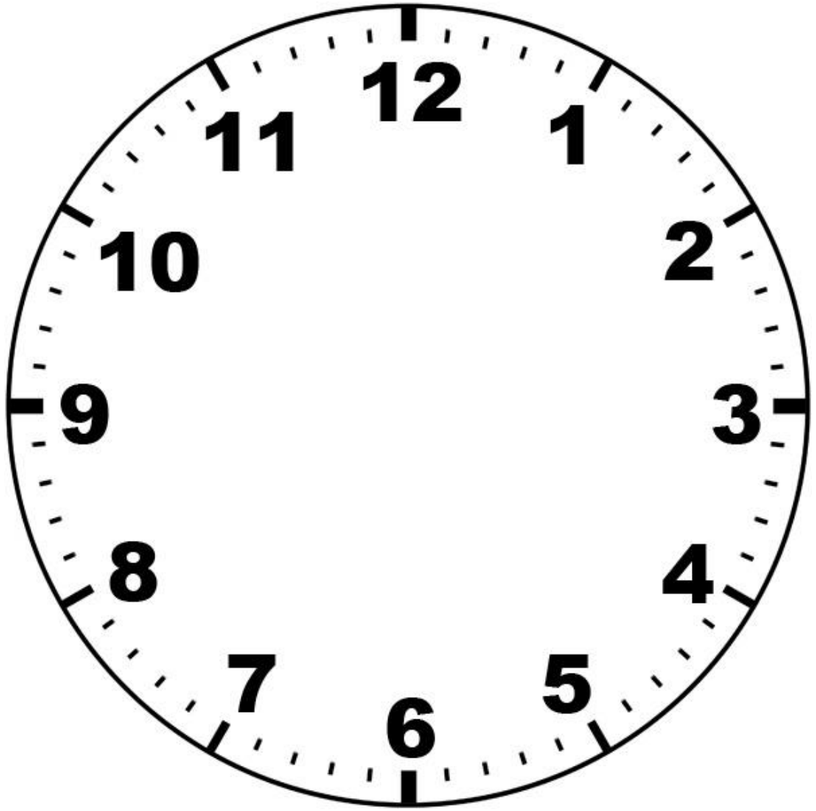
5, 6, **7**

Which answer?

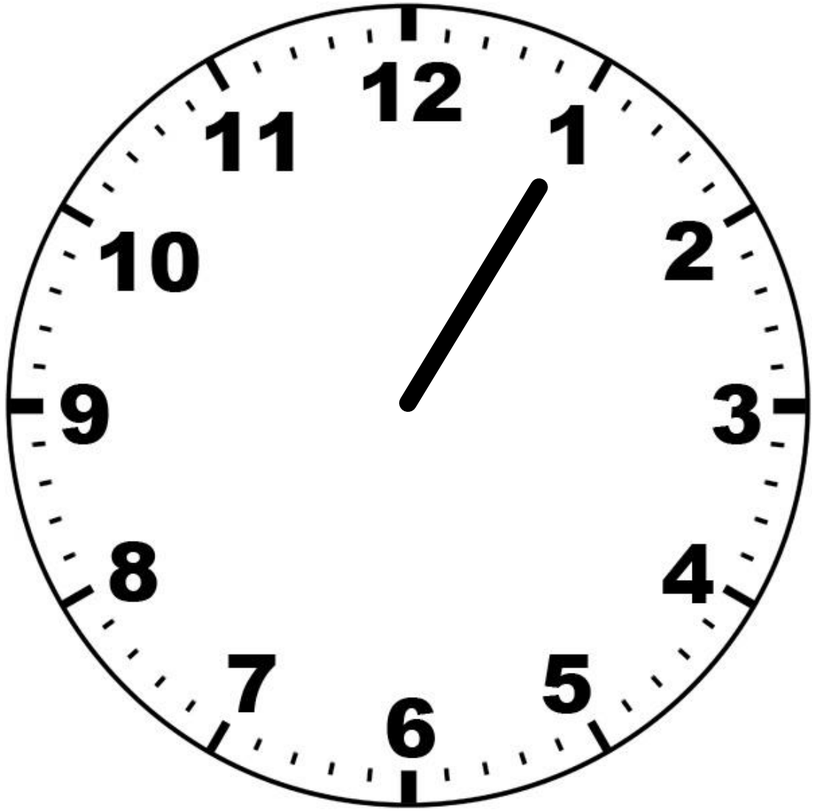
How long is the pencil crayon?

8cm

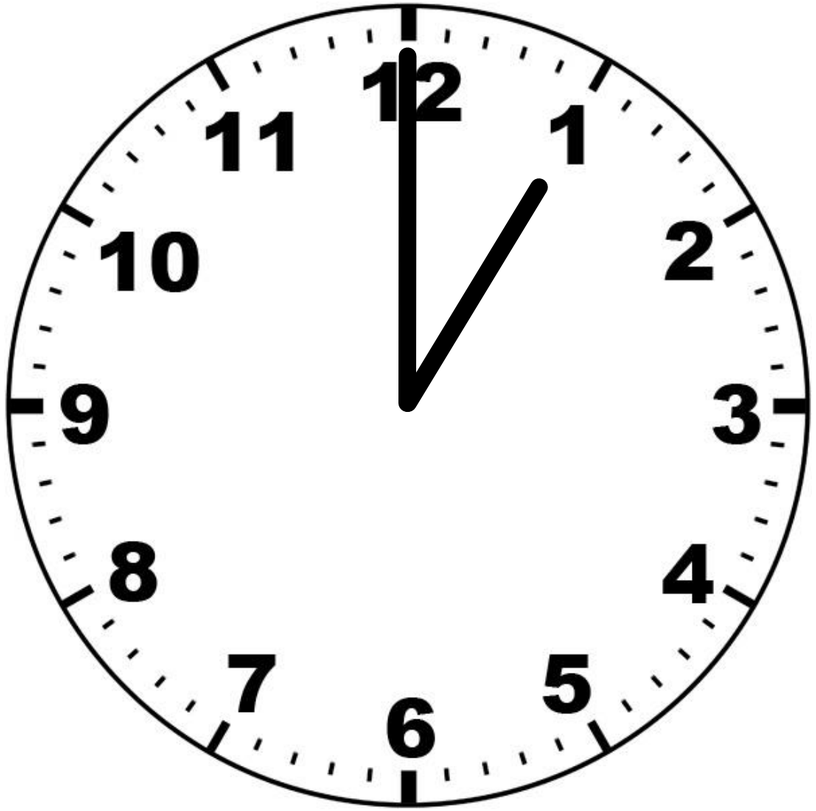
10cm



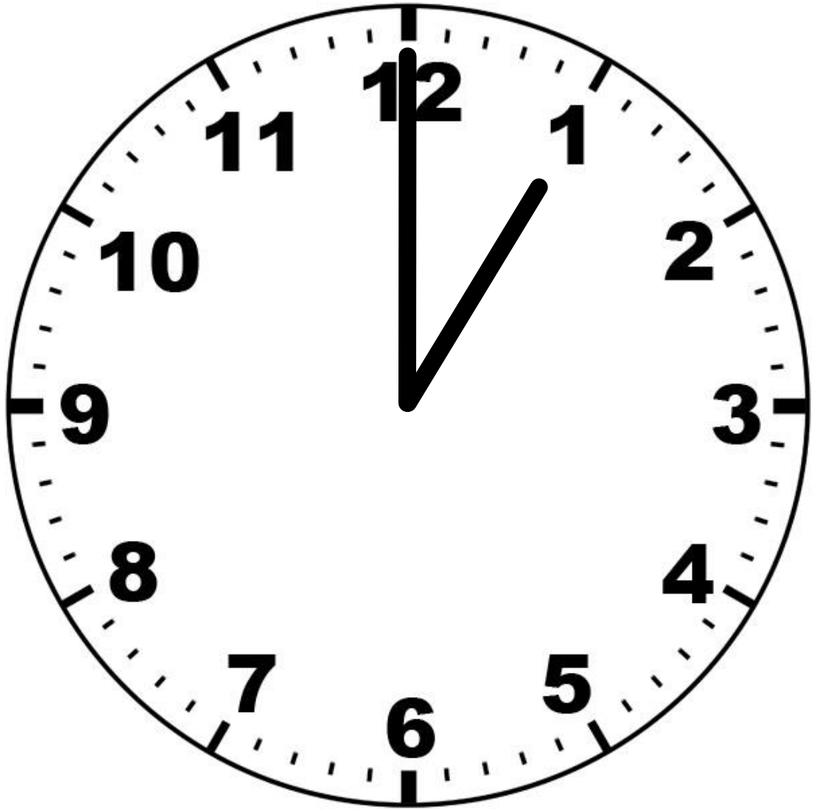
1:00



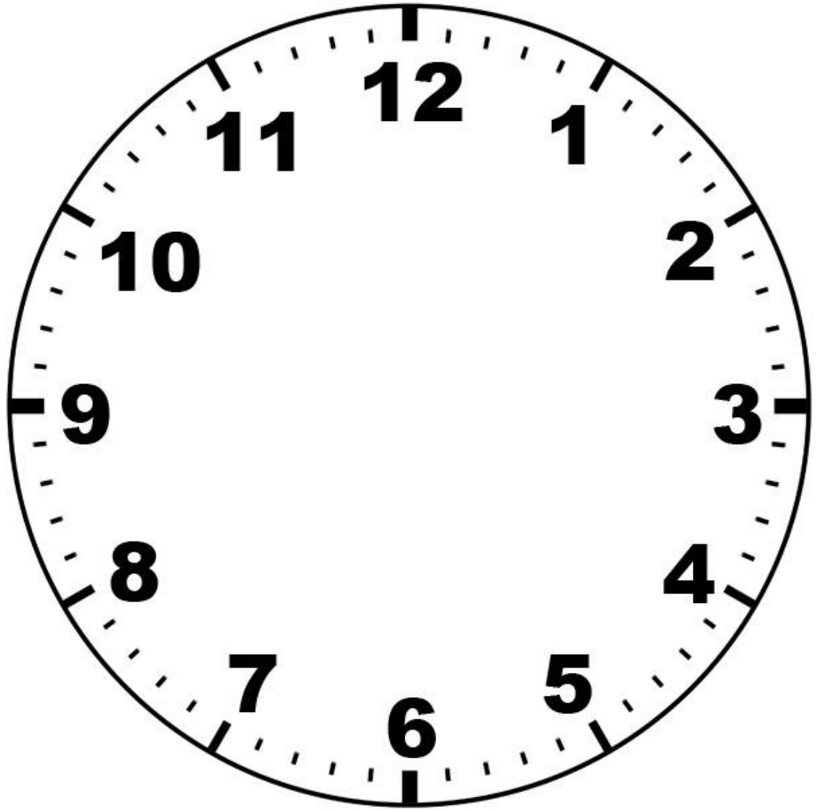
1:00



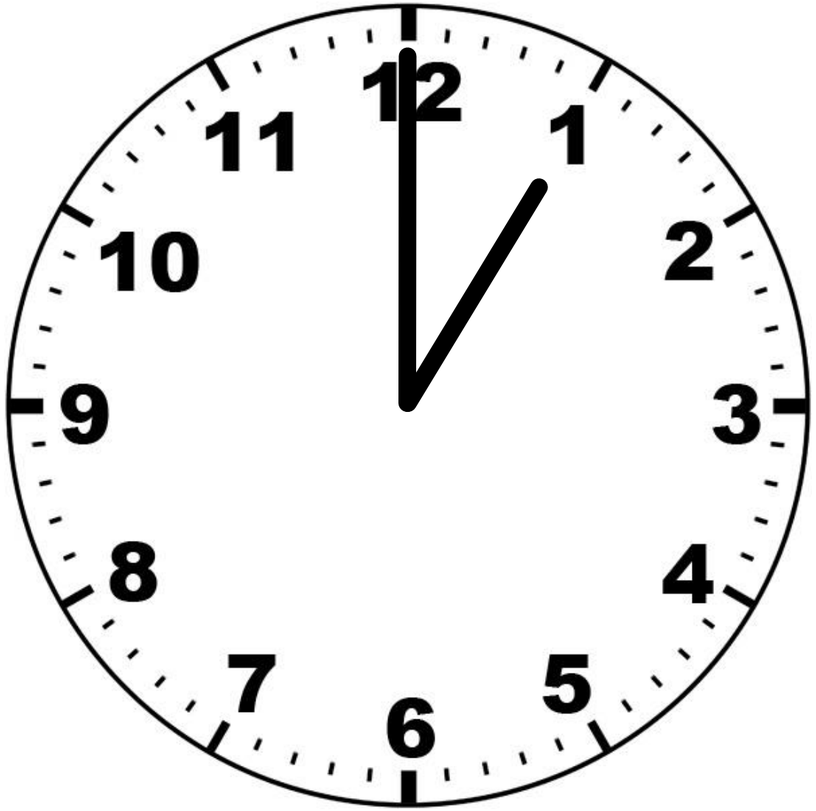
1:00



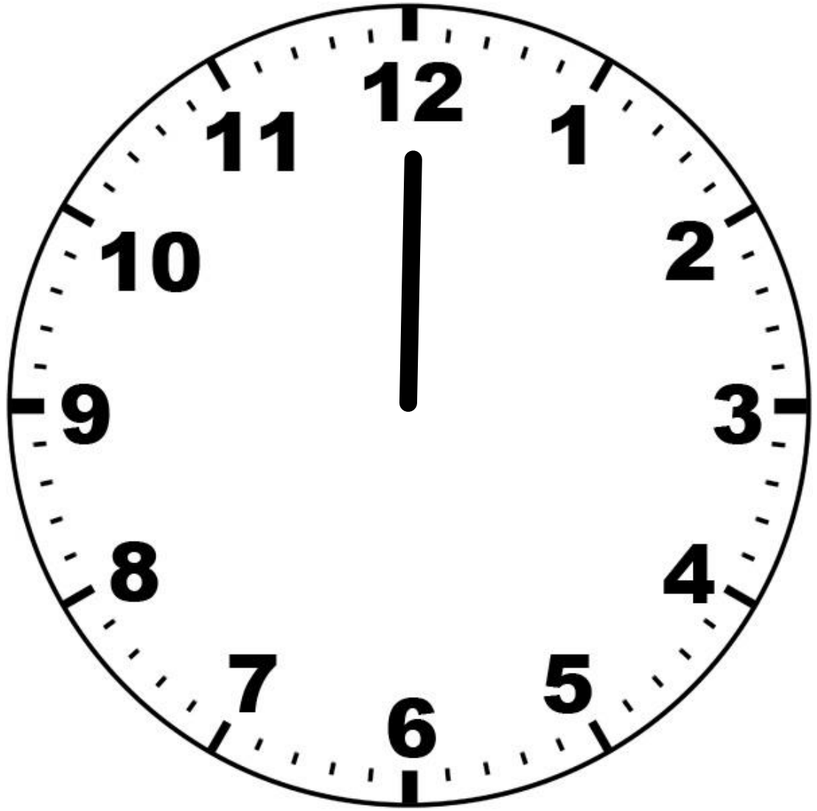
1:00



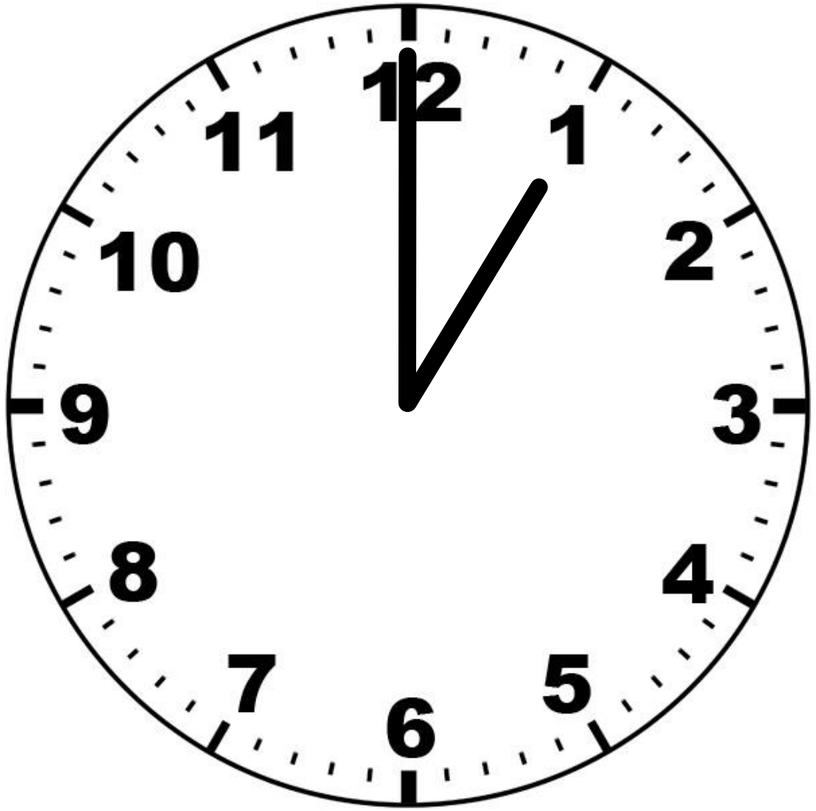
12:05



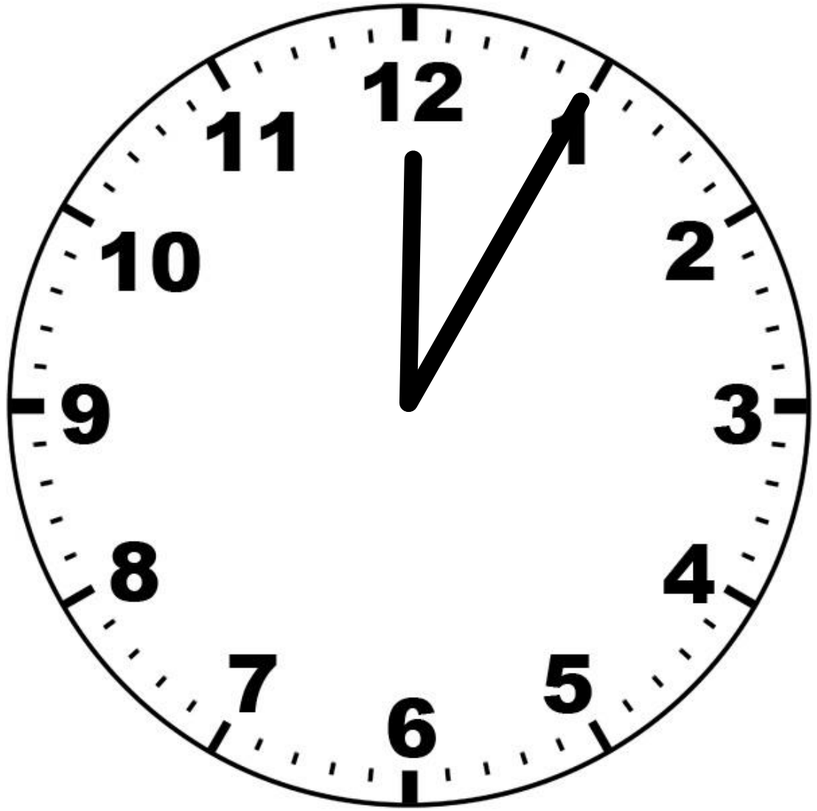
1:00



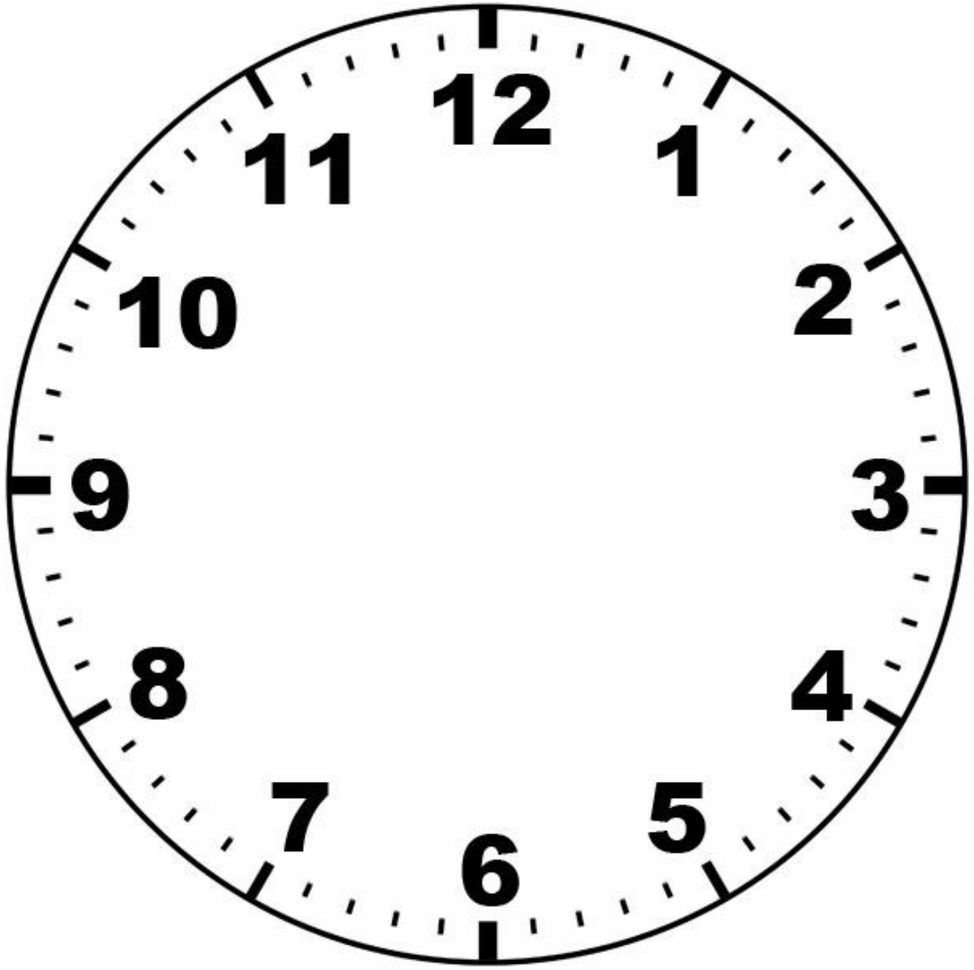
12:05



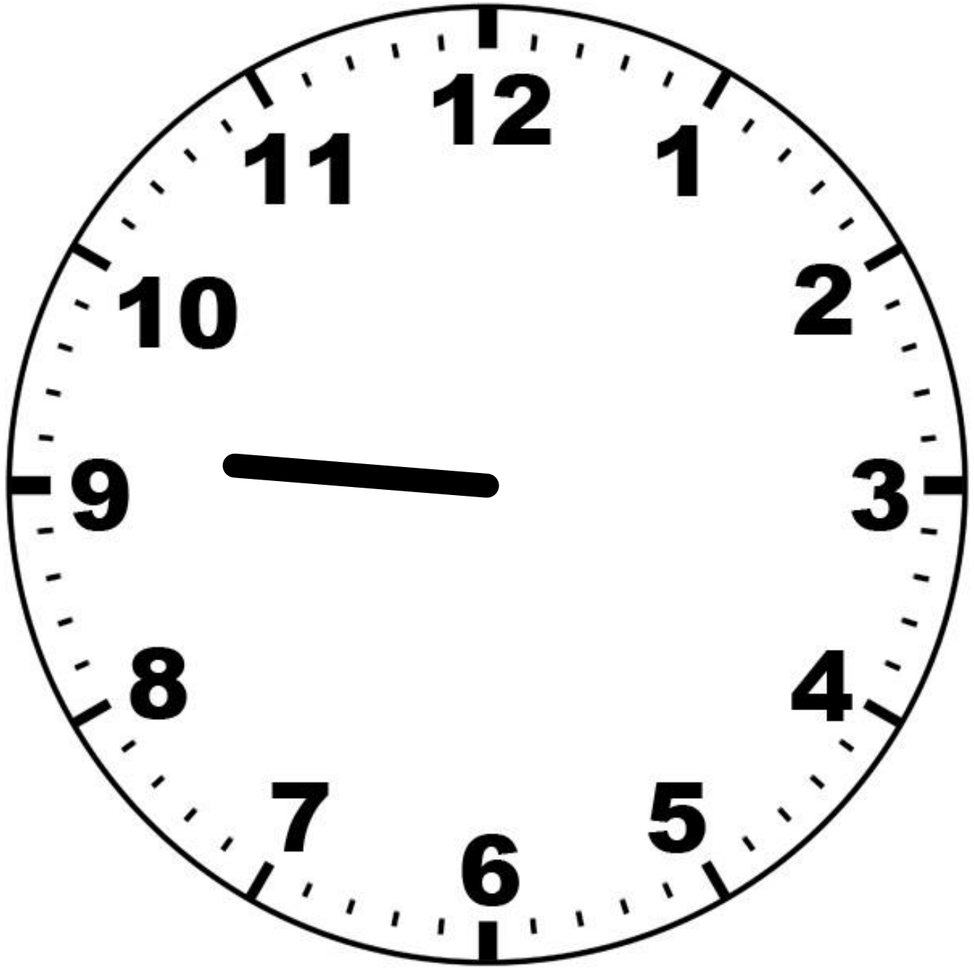
1:00



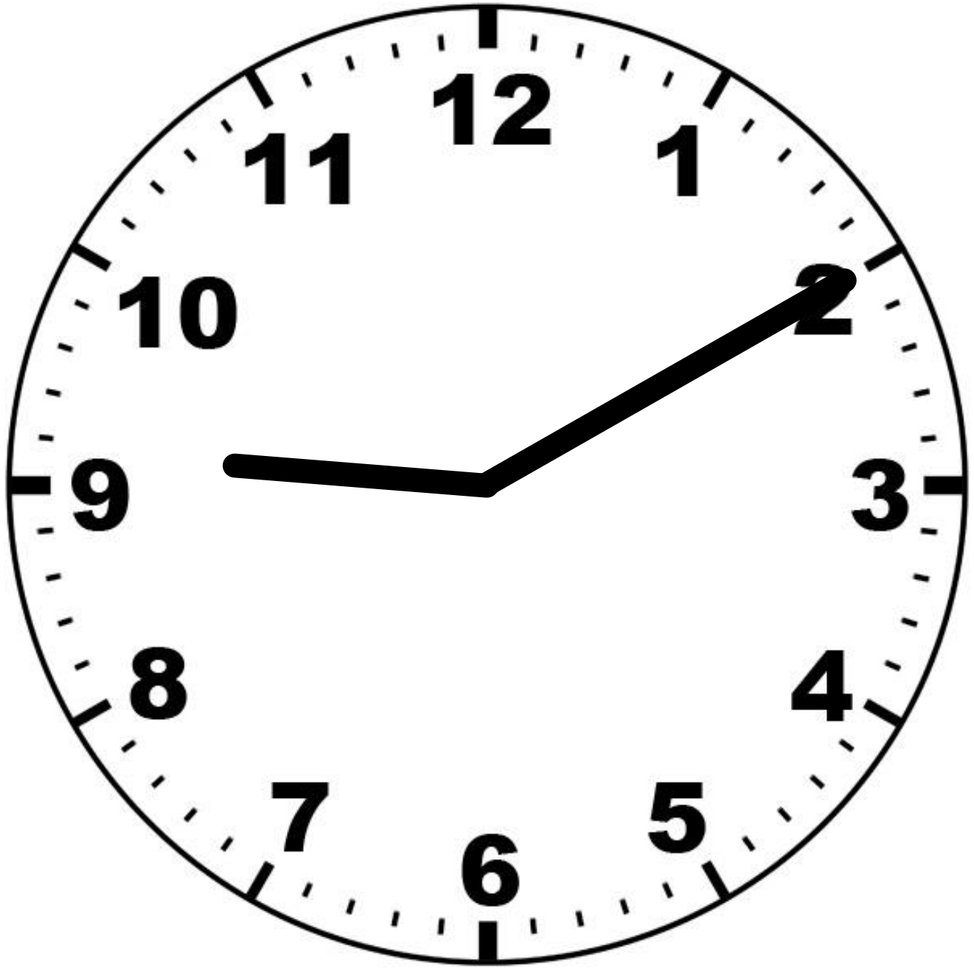
12:05



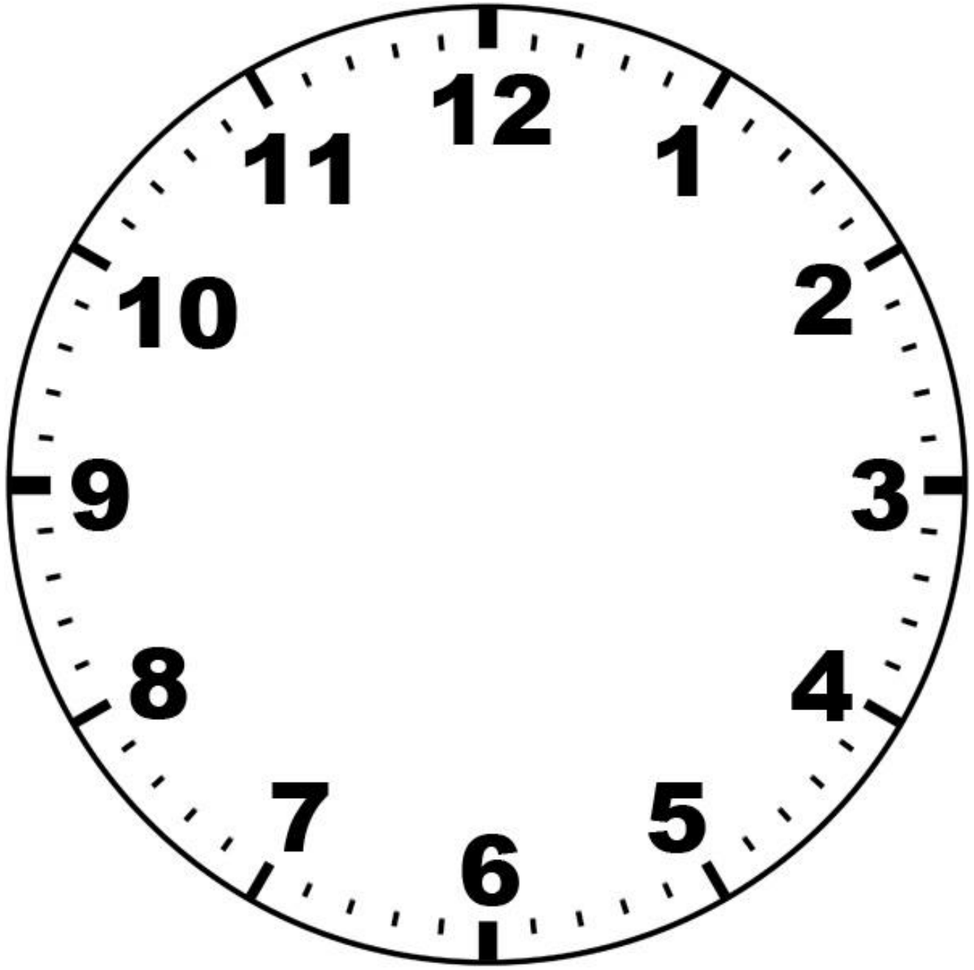
9:10



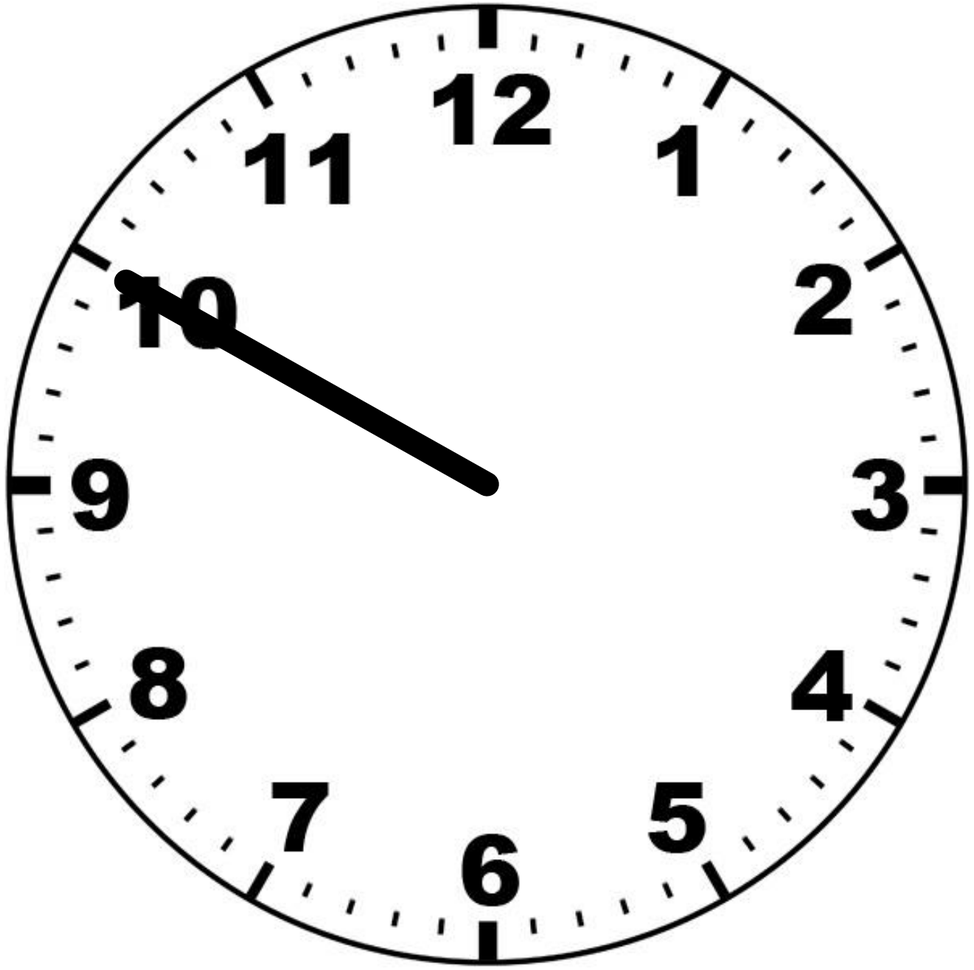
9:10



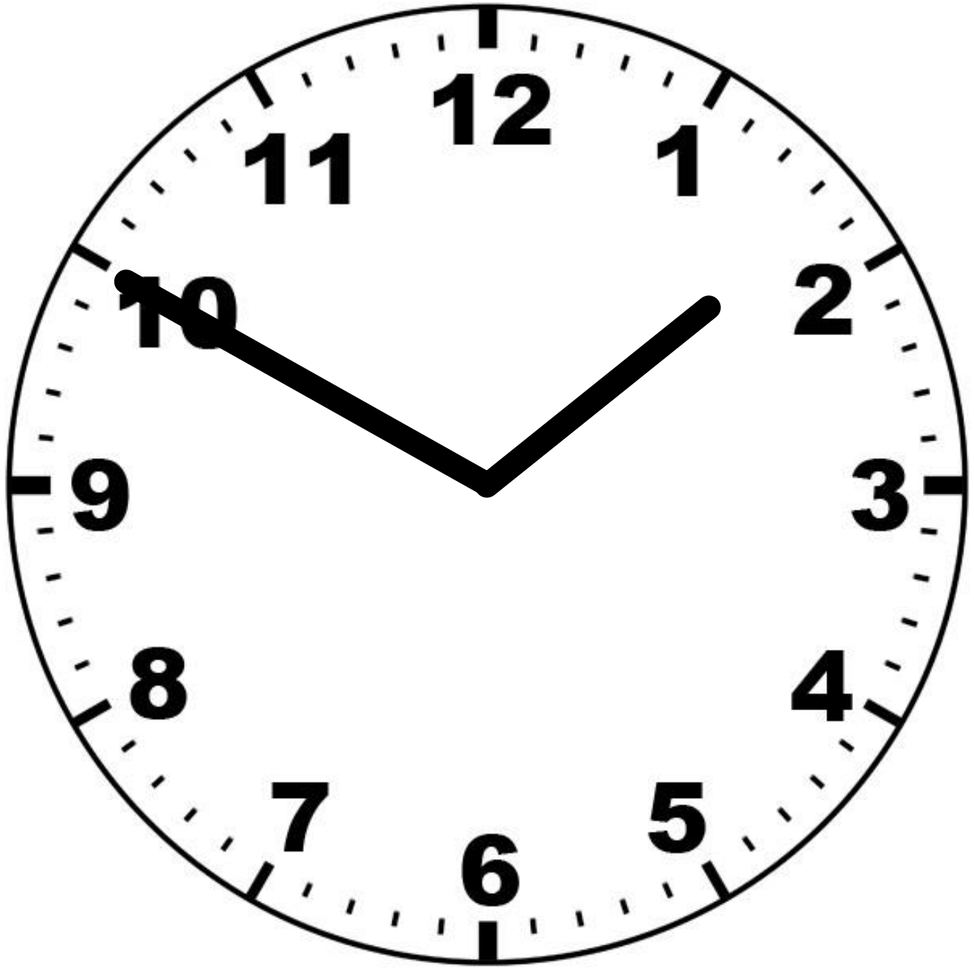
9:10



1:50

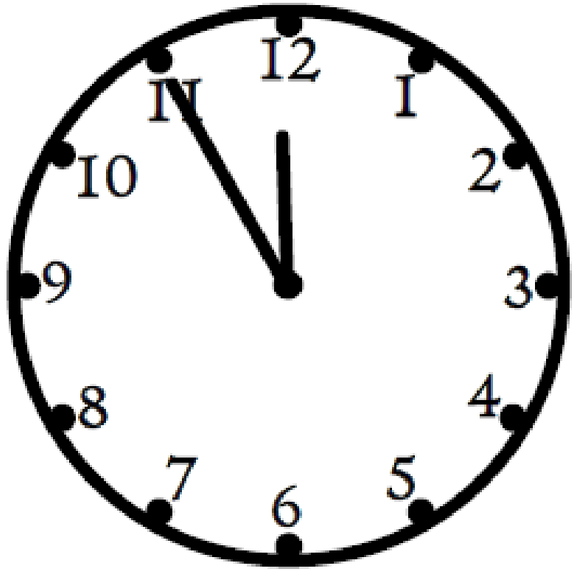


1:50

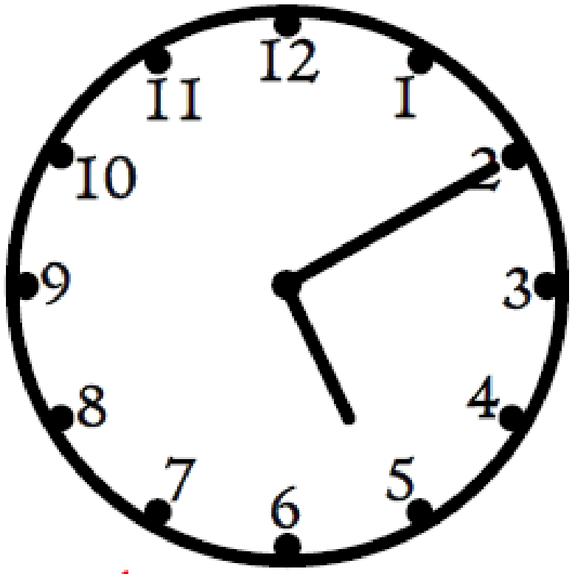


1:50

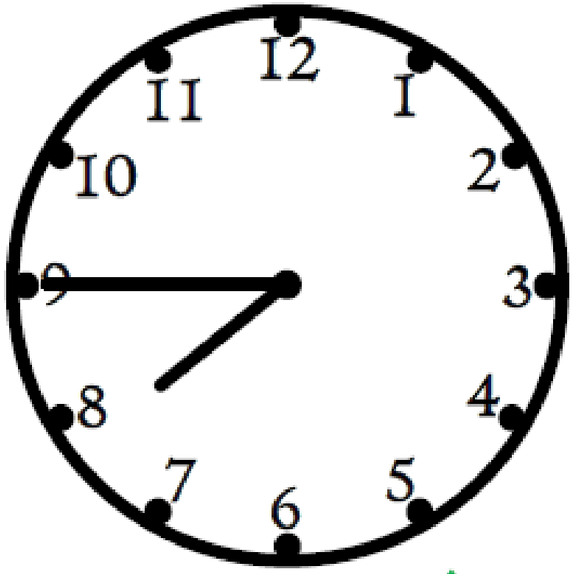
Explain the Mistakes



11:00



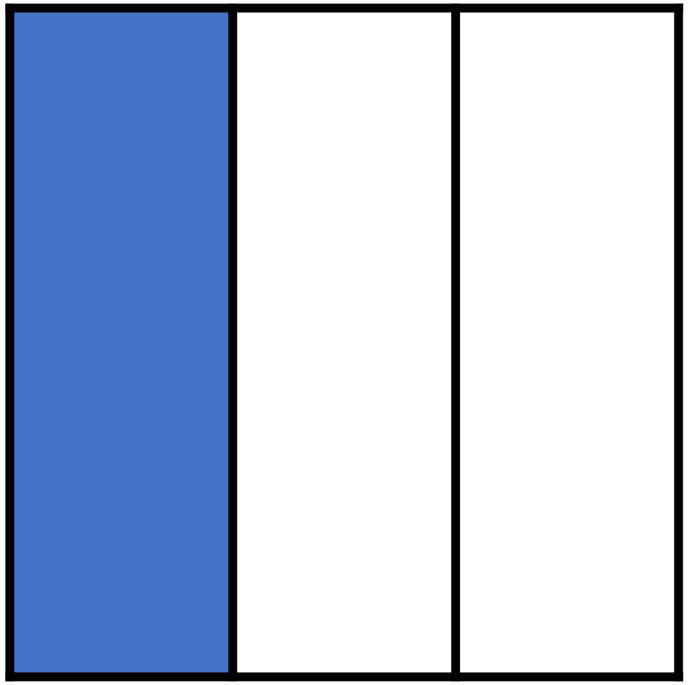
5:02



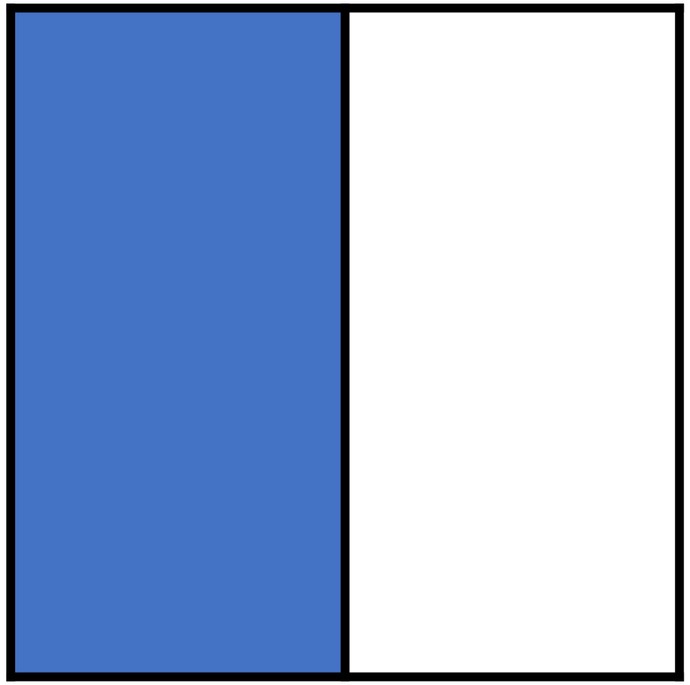
8:45

When the minute hand points to… it means…

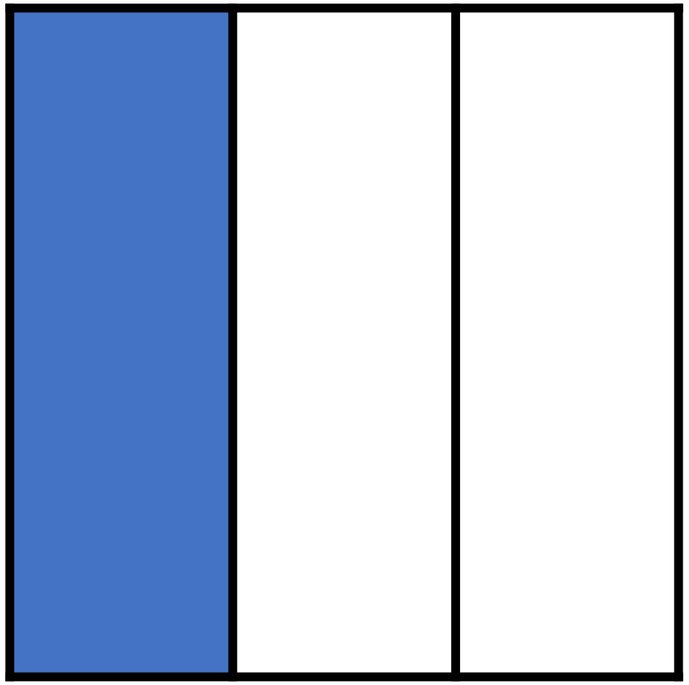
If the time was… the hour hand would be…



$$\frac{1}{2}$$

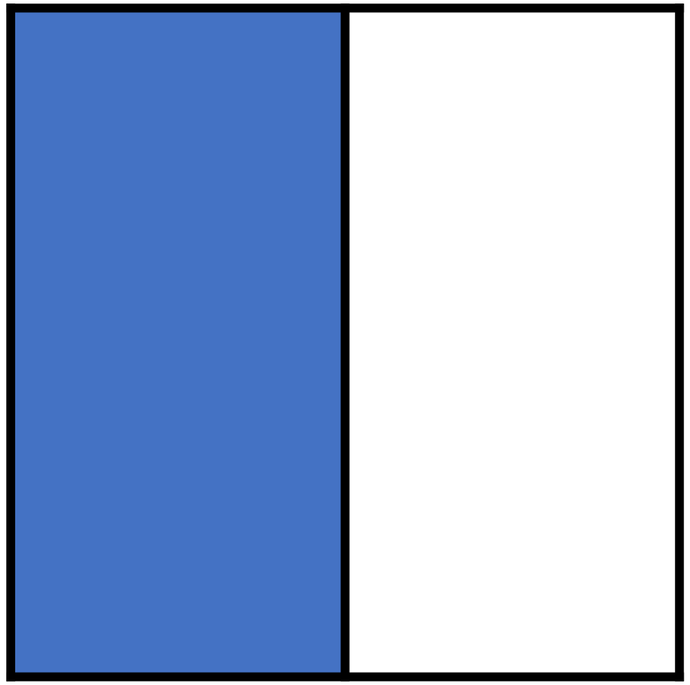


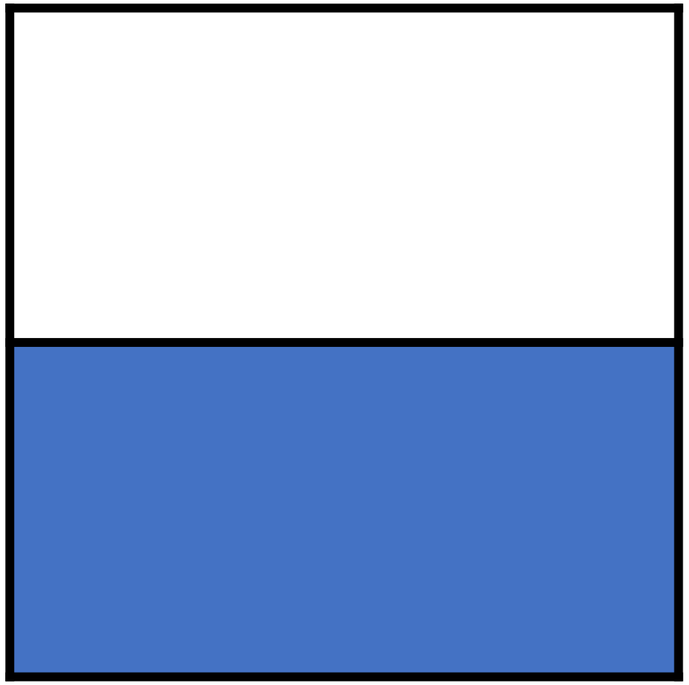
✗



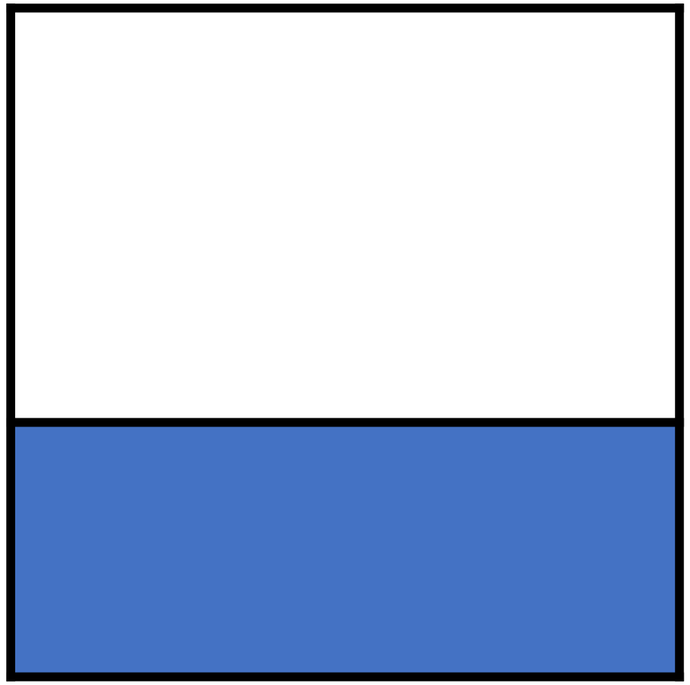
$$\frac{1}{2}$$

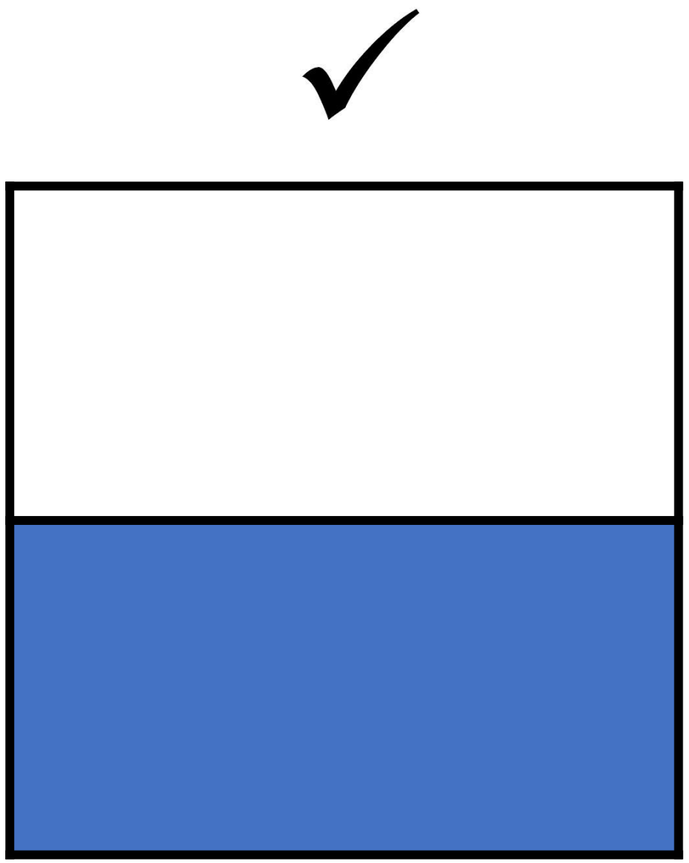
✓



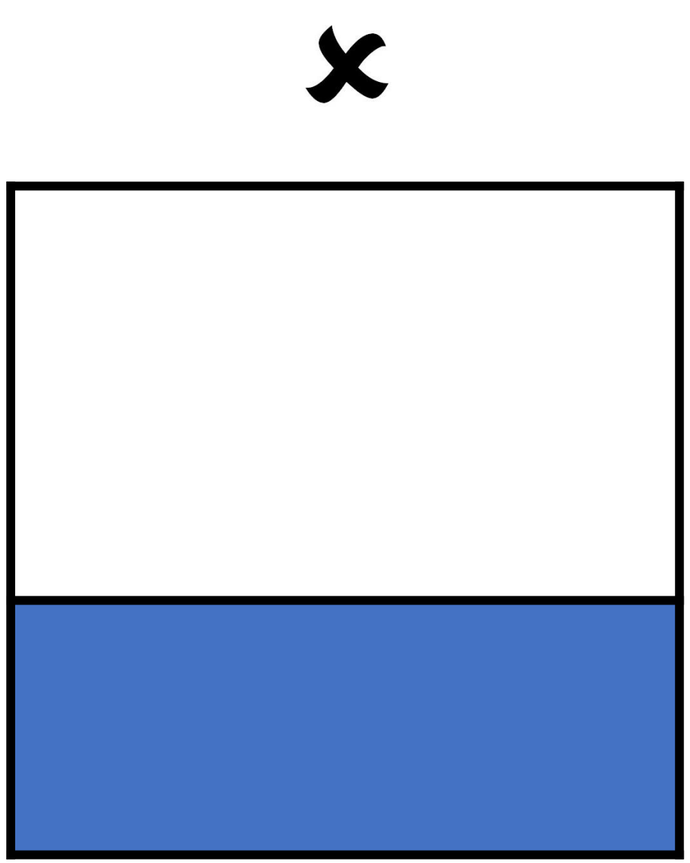


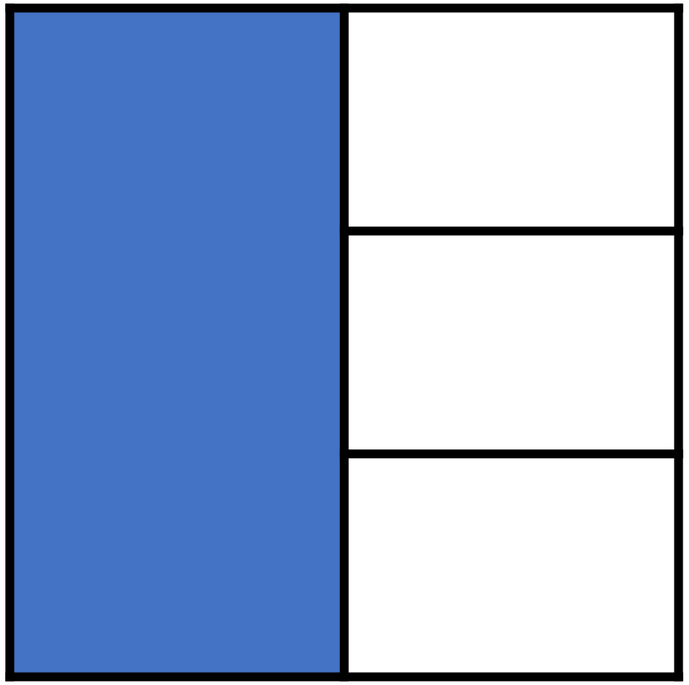
$$\frac{1}{2}$$



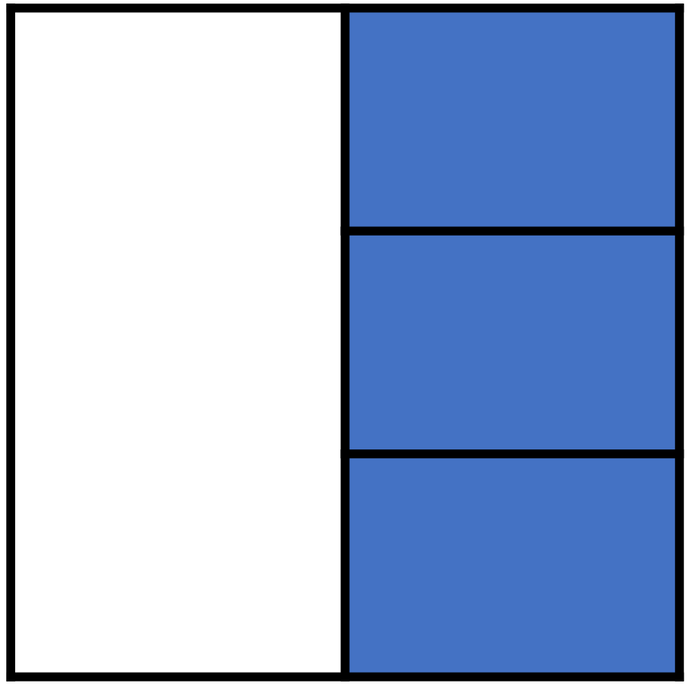


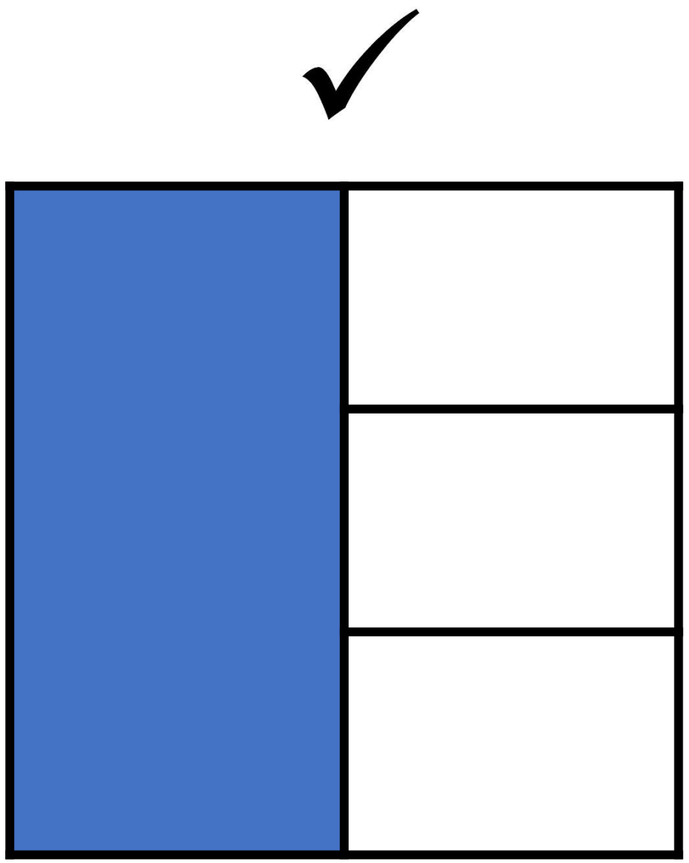
$$\frac{1}{2}$$



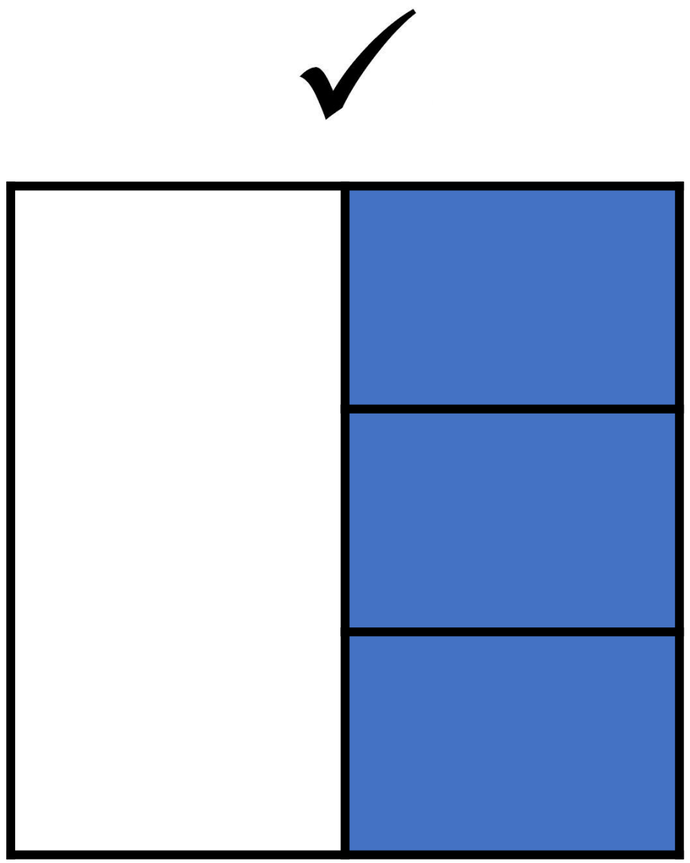


$$\frac{1}{2}$$



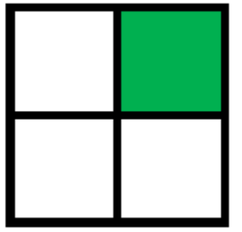


$$\frac{1}{2}$$

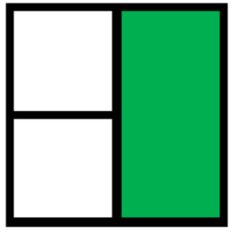


Explain the Mistakes

$\frac{1}{4}$ green



$\frac{1}{3}$ green

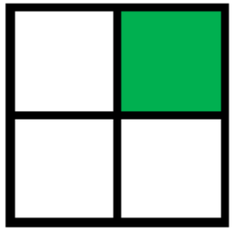


$\frac{1}{3}$ green

$$\frac{\text{numerator}}{\text{denominator}}$$

Explain the Mistakes

$\frac{1}{4}$ green



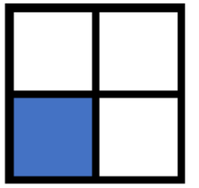
$\frac{1}{3}$ green

$\frac{1}{3}$ green

$$\frac{\text{numerator}}{\text{denominator}}$$

Correct or Incorrect? ✓ or ✗

What fraction of each square is blue?



$\frac{1}{4}$

$\frac{1}{3}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{3}{1}$

$\frac{3}{4}$

$\frac{1}{3}$

$\frac{1}{3}$

For the incorrect examples, explain the mistakes.

20 21 22 23 24 25

$$I = 1$$

$$V = 5$$

$$X = 10$$

20	21	22	23	24	25
XX					

$$I = 1$$

$$V = 5$$

$$X = 10$$

20	**21**	**22**	**23**	**24**	**25**
XX	XXI	XXII	XXIII		

I = 1

V = 5

X = 10

20	**21**	**22**	**23**	**24**	**25**
XX	XXI	XXII	XXIII		XXV

I = 1

V = 5

X = 10

20	**21**	**22**	**23**	**24**	**25**
XX	XXI	XXII	XXIII	XXIV	XXV

I = 1

V = 5

X = 10

Explain the Mistakes

In Roman Numerals, 4 is written IIII

XL is 60 because it is 10 + 50

In Roman Numerals, 15 is written VVV

I = 1

V = 5

X = 10

L = 50

Explain the Mistakes

In Roman Numerals, 4 is written IIII

XL is 60 because it is 10 + 50

In Roman Numerals, 15 is written VVV

I = 1
V = 5
X = 10
L = 50

Which Answer?

What is the missing Roman Numeral or number?

9 = ☐ XI IX	14 = ☐ XIIII XIV	21 = ☐ XIX XXI
XVI = ☐ 14 16	XIX = ☐ 19 21	XVIII = ☐ 17 18

I = 1
V = 5
X = 10

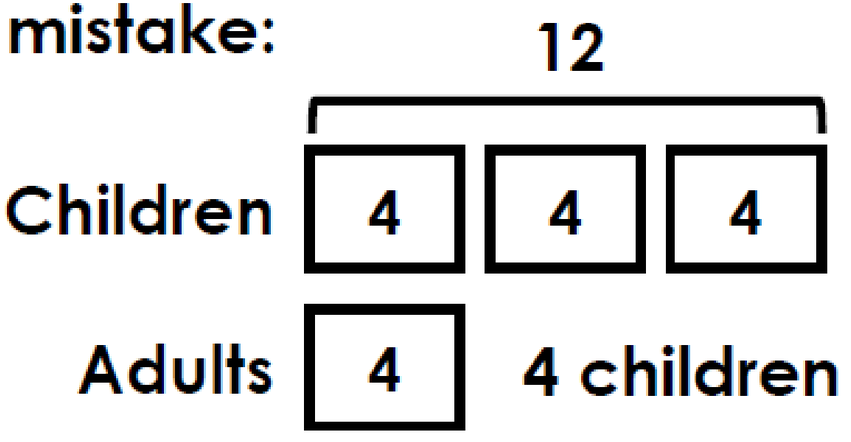
Explain the mistakes.

Explain the Mistake

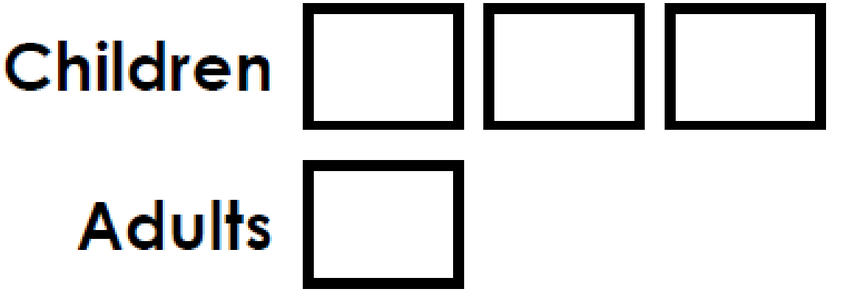
For every three children on the school trip there is one adult. There are 12 more children than adults on the trip.

How many adults went on the school trip?

Explain the mistake:



Do the drawing correctly:

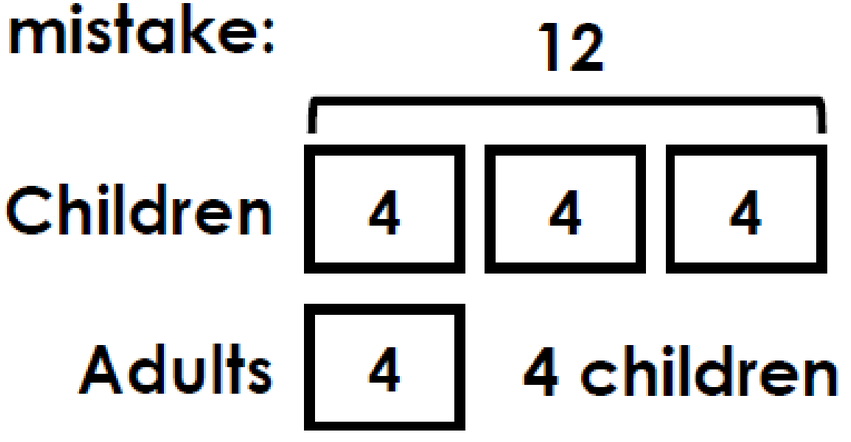


Explain the Mistake

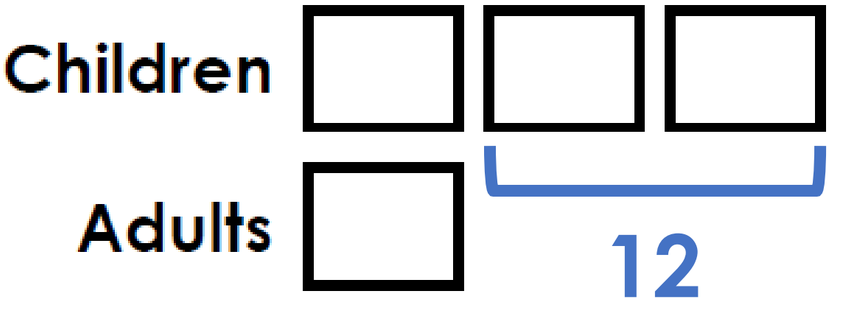
For every three children on the school trip there is one adult. There are 12 more children than adults on the trip.

How many adults went on the school trip?

Explain the mistake:



Do the drawing correctly:

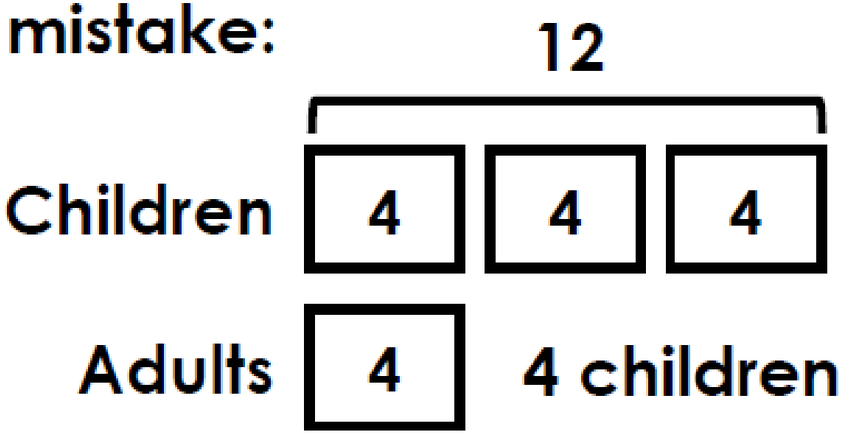


Explain the Mistake

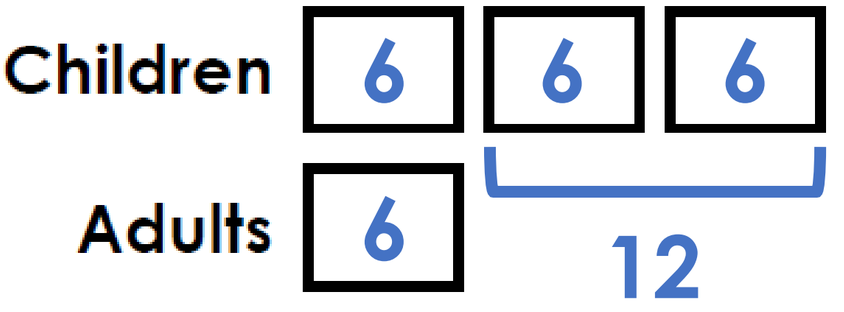
For every three children on the school trip there is one adult. There are 12 more children than adults on the trip.

How many adults went on the school trip?

Explain the mistake:



Do the drawing correctly:

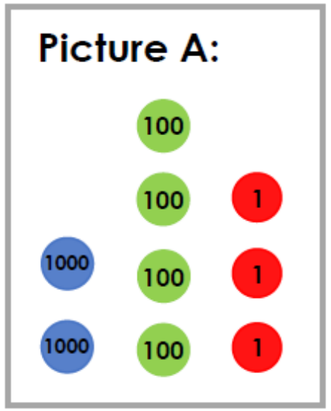


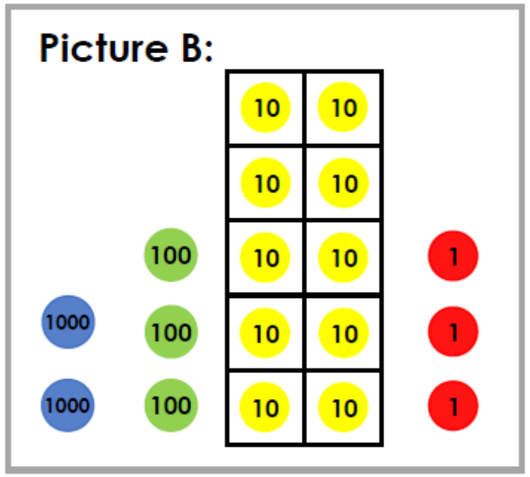
Show understanding in different ways

Use of representations

Read the Pictures

Part 1: Explain how **both** pictures show **2403**





Part 2: Which picture is **most helpful** for calculating **2403 – 10?**

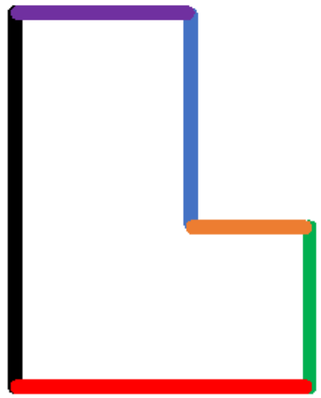
Explain why.

Correct or Incorrect?

✓ or ✗

- ☐ blue = green + red + yellow
- ☐ blue – red = green + yellow
- ☐ blue + yellow = red + green
- ☐ blue – green = red + yellow

Explain



Complete the sentences:

purple + orange =

black – blue =

Write your own.

Make logical leaps

I know… so…

See relationships, generalise

Small Difference Questions

I know… so…

32 ÷ 8 = 4

38 ÷ 8 =

50 ÷ 6 = 8 r 2

44 ÷ 6 =

24 ÷ 3 = 8

48 ÷ 6 =

30 ÷ 4 = 7 r 2

38 ÷ 4 =

37 ÷ 3 = 12 r 1

39 ÷ 3 =

14 ÷ 4 = 3 r 2

54 ÷ 4 =

Make logical leaps

I know… so…

See relationships, generalise

Small Difference Questions

I know… so…

32 ÷ 8 = 4	**50 ÷ 6 = 8 r 2**	**24 ÷ 3 = 8**
38 ÷ 8 = **5 r 2**	44 ÷ 6 =	48 ÷ 6 =
30 ÷ 4 = 7 r 2	**37 ÷ 3 = 12 r 1**	**14 ÷ 4 = 3 r 2**
38 ÷ 4 = **9 r 2**	39 ÷ 3 =	54 ÷ 4 =

Make logical leaps

I know… so…

See relationships, generalise

Small Difference Questions

I know… so…

32 ÷ 8 = 4

38 ÷ 8 = **5 r 2**

50 ÷ 6 = 8 r 2

44 ÷ 6 = **5 r 2**

24 ÷ 3 = 8

48 ÷ 6 =

30 ÷ 4 = 7 r 2

38 ÷ 4 = **9 r 2**

37 ÷ 3 = 12 r 1

39 ÷ 3 = **9 r 2**

14 ÷ 4 = 3 r 2

54 ÷ 4 =

Make logical leaps

I know… so…

See relationships, generalise

Small Difference Questions

I know… so…

32 ÷ 8 = 4

38 ÷ 8 = **5 r 2**

50 ÷ 6 = 8 r 2

44 ÷ 6 = **5 r 2**

24 ÷ 3 = 8

48 ÷ 6 = **8**

30 ÷ 4 = 7 r 2

38 ÷ 4 = **9 r 2**

37 ÷ 3 = 12 r 1

39 ÷ 3 = **9 r 2**

14 ÷ 4 = 3 r 2

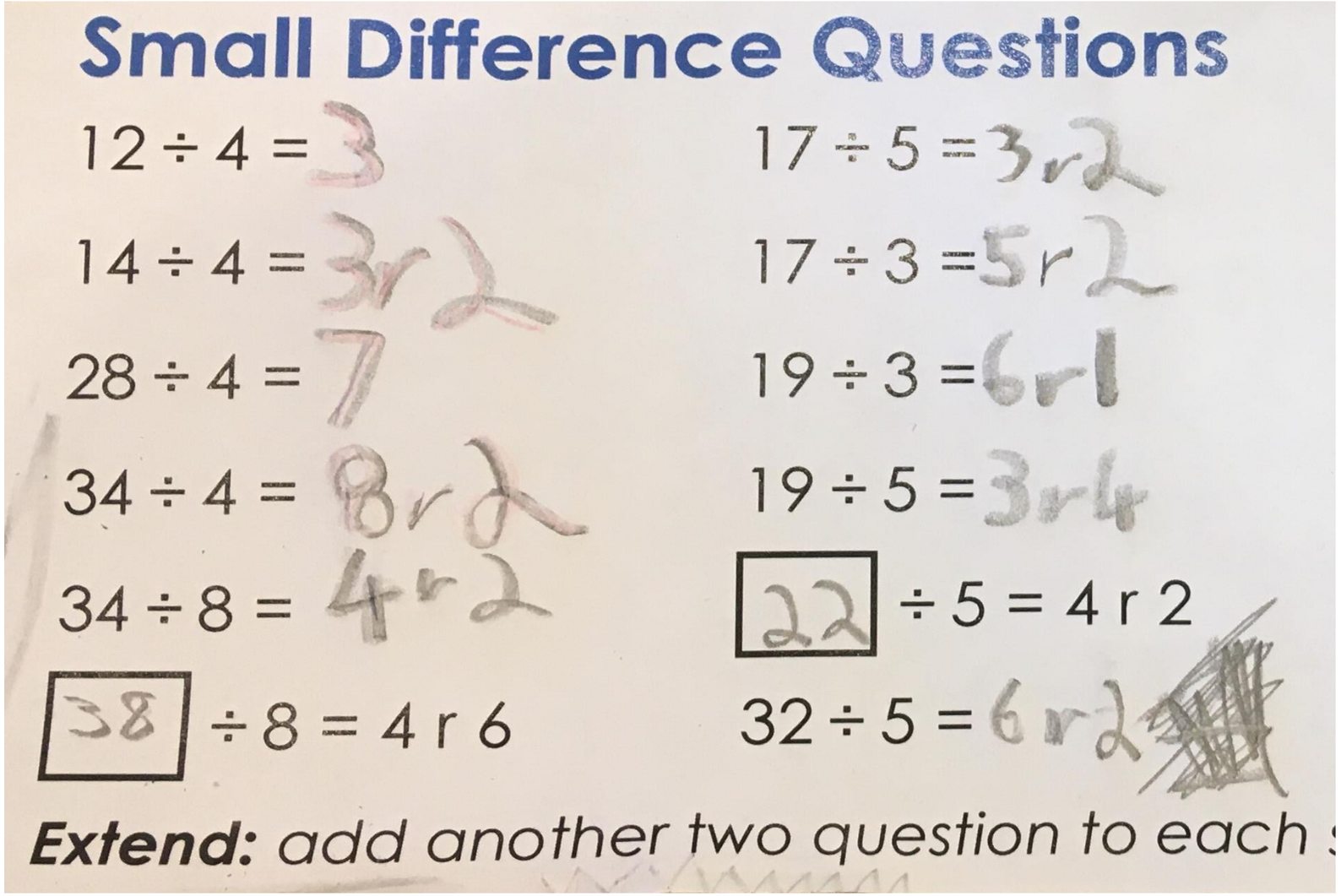
54 ÷ 4 = **13 r 2**

Make logical leaps

I know… so…

See relationships, generalise

Small Difference Questions



Small Difference Questions

12 ÷ 4 = 3

14 ÷ 4 = 3r2

28 ÷ 4 = 7

34 ÷ 4 = 8r2

34 ÷ 8 = 4r2

38 ÷ 8 = 4 r 6

17 ÷ 5 = 3r2

17 ÷ 3 = 5r2

19 ÷ 3 = 6r1

19 ÷ 5 = 3r4

22 ÷ 5 = 4 r 2

32 ÷ 5 = 6r2

Extend: *add another two question to each*

Small Difference Questions

6 + 4 = 10

6 + 6 = 12

7 + 5 = 12

17 + 5 = 22

5 + 17 = 22

4 + 4 = 8

8 + 8 = 16

8 + 7 = 15

9 + 6 = 15

9 + 16 = 25

I know... so...

16 × 7 = 112

17 × 7 = 119

12 × 10 = 120

12 × 9 = 108

7 × 6 = 42

7 × 12 = 84

12 × 6 = 72

12 × 8 = 96

8 × 6 = 48

80 × 6 = 480

15 × 8 = 120

15 × 7 = 105

I know… so…

16 × 7 = 112

17 × 7 = 119

12 × 10 = 120

12 × 9 = 108

7 × 6 = 42

7 × 12 = 84

12 × 6 = 72

12 × 8 = 96

8 × 6 = 48

80 × 6 = 480

15 × 8 = 120

15 × 7 = 105

Small Difference Questions

7 × 3 = 21

3 × 7 = 21

6 × 7 = 42

12 × 7 = 84

12 × 5 = 60

…is the same as…

2 × 8 = 16

4 × 8 = 32

+2 6 × 8 = 48

12 × 4 = 48

+2 14 × 4 = 56

…is double…

5 × 7 = 35

10 × 7 = 70

7 × 10 = 70

7 × 9 = 63

14 × 9 = 126

… is 7 less than…

-5	-3	-1	1	3	5

-5	-3	-1	1	3	5

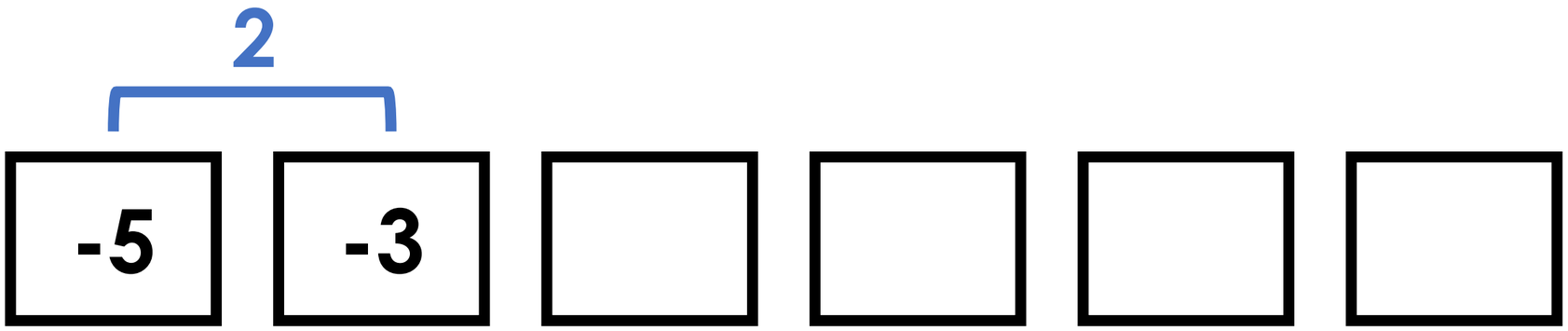
-5	-1	3	7	11	15

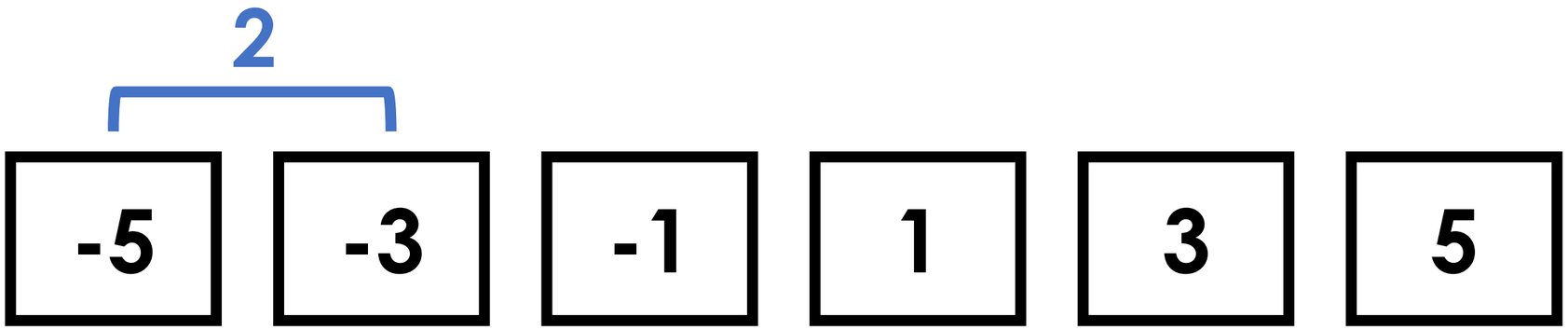
-5	-3	-1	1	3	5

-5	-1	3	7	11	15

-5	1	7	13	19	25

-5	-3				

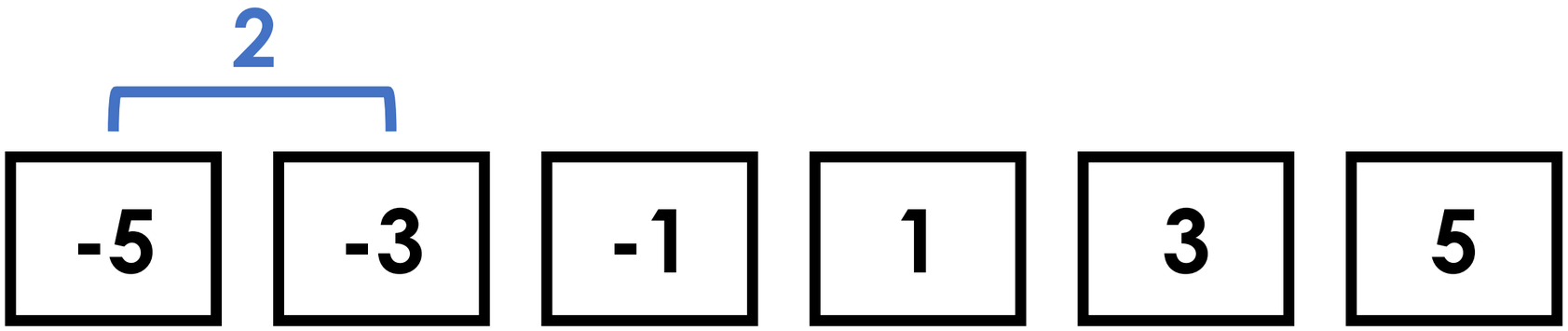

2
-5
-3

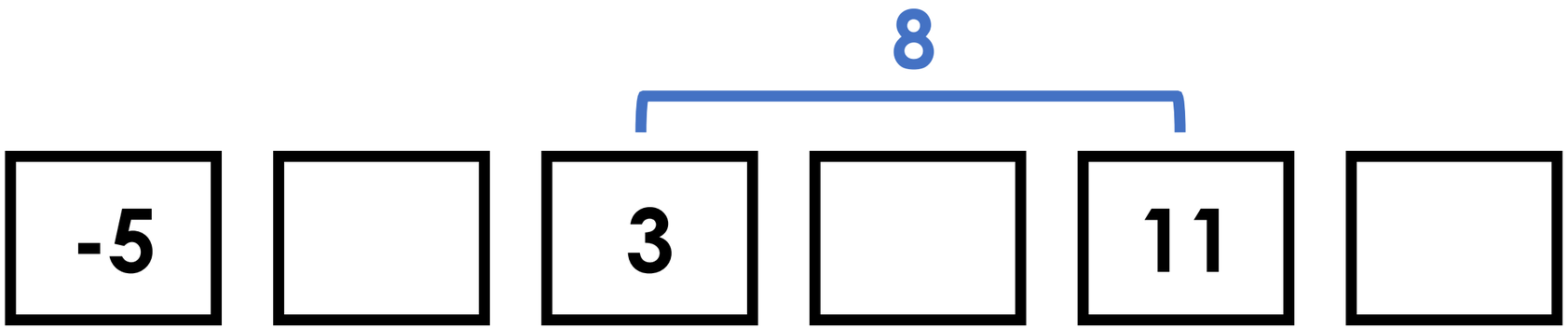

2
-5
-3
-1
1
3
5

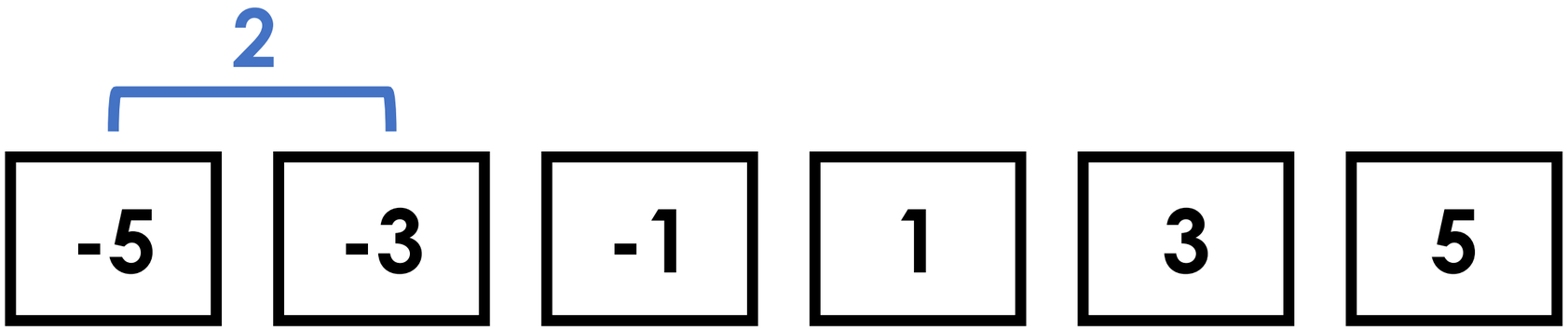
2

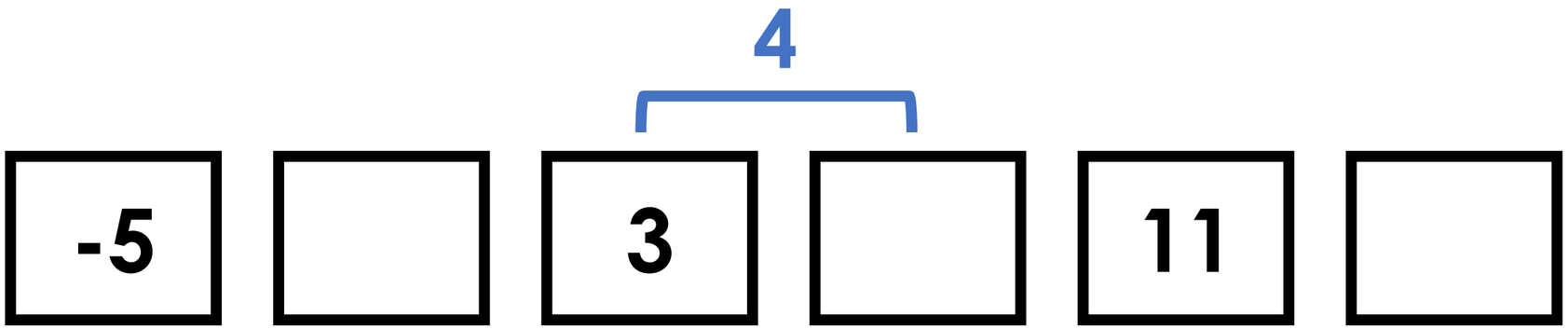
-5	-3	-1	1	3	5

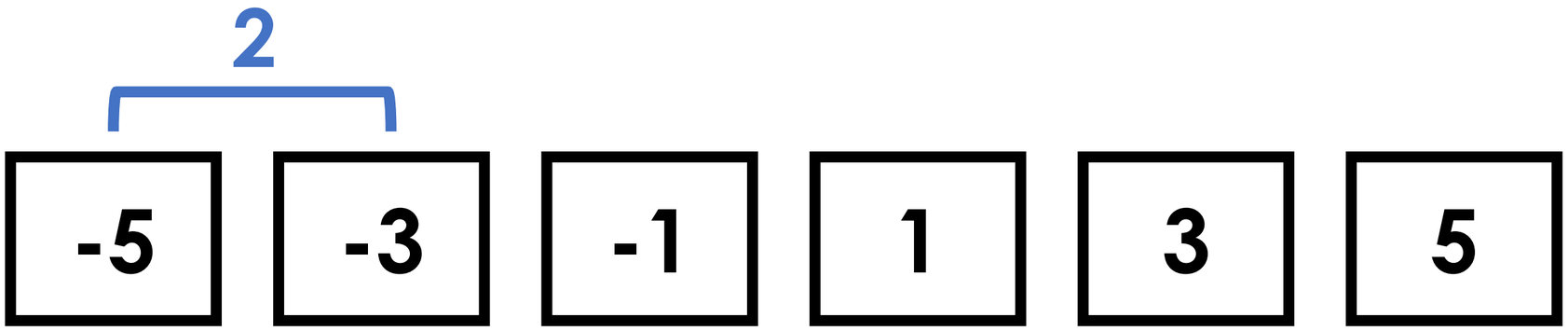
-5		3		11	

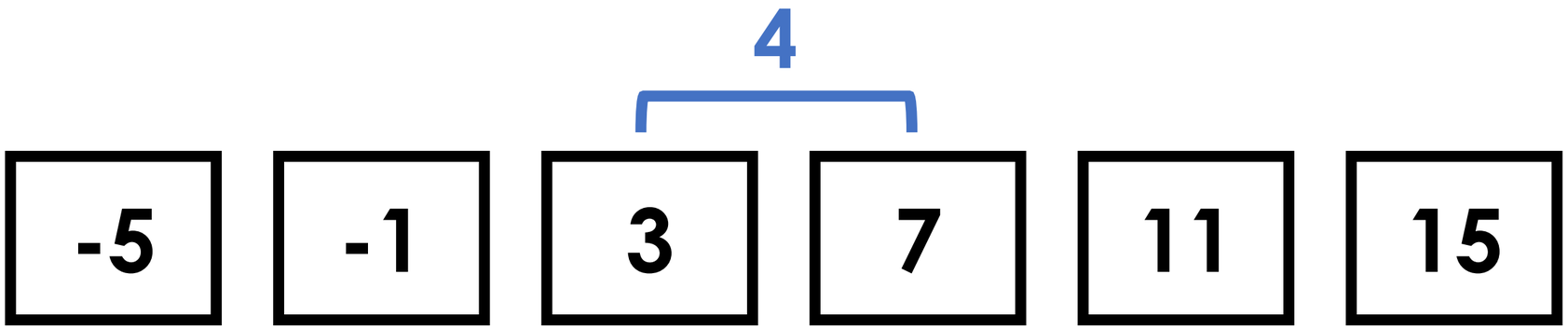

2
-5
-3
-1
1
3
5

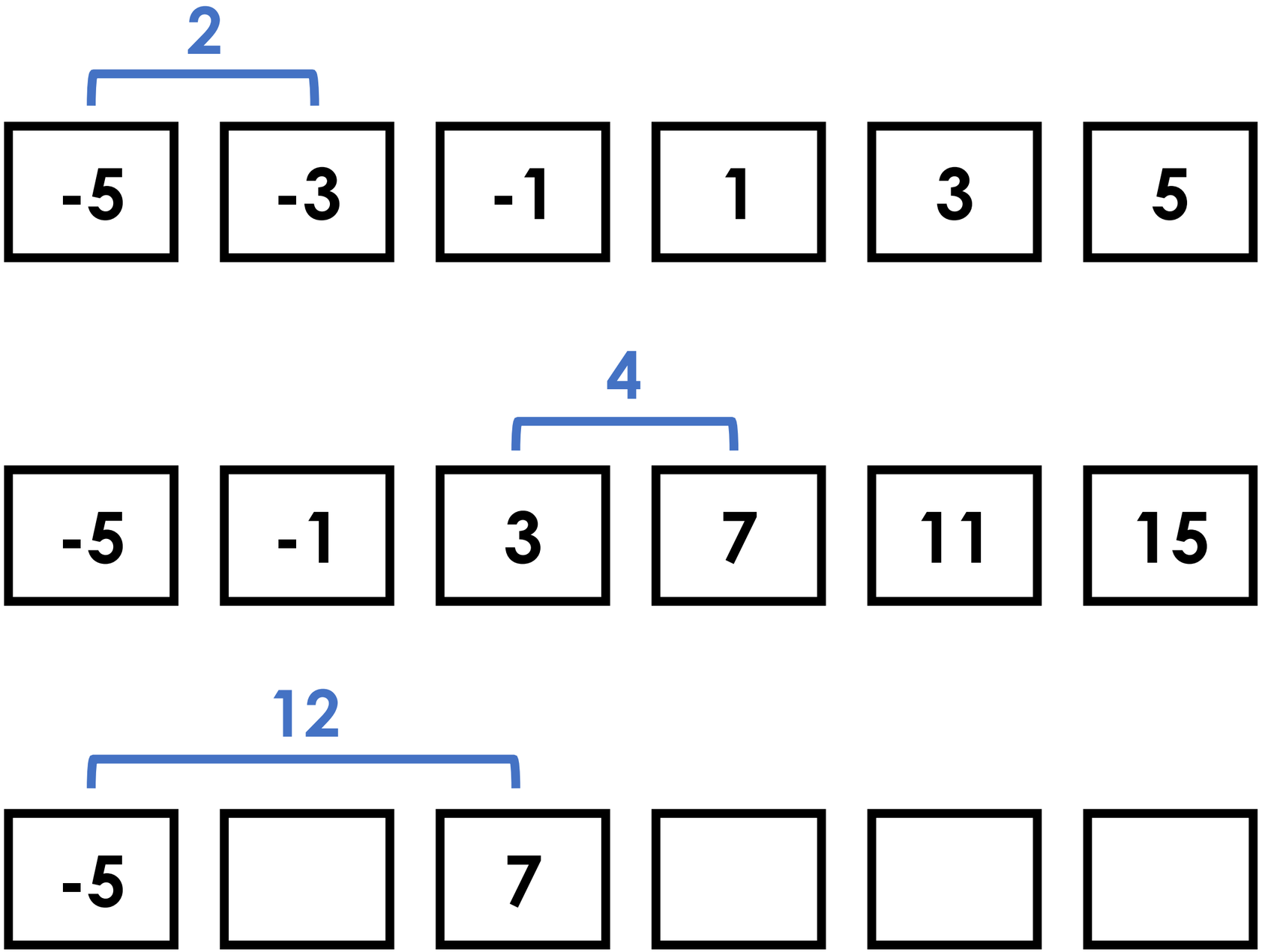

8
-5
3
11

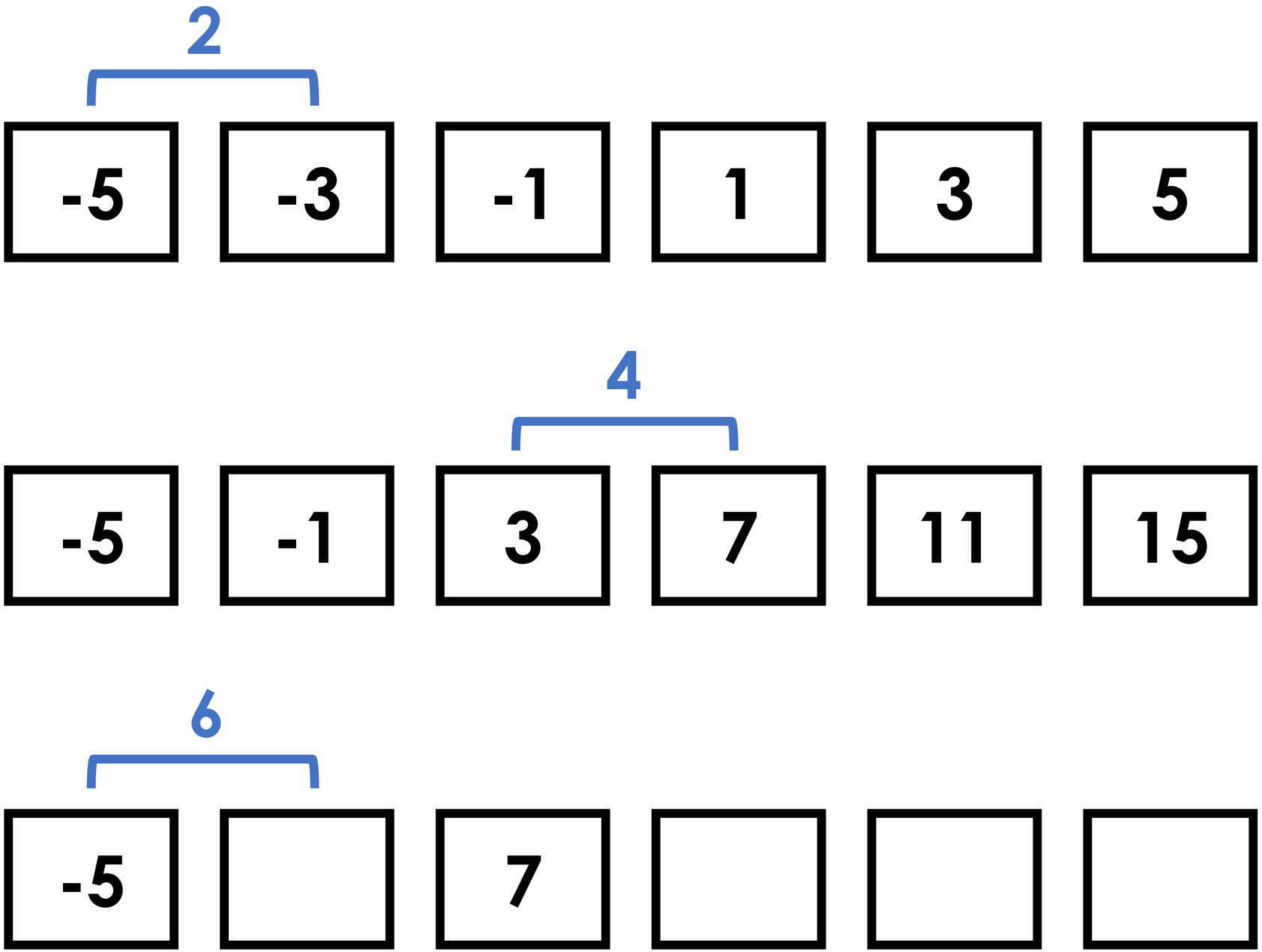

2
-5
-3
-1
1
3
5

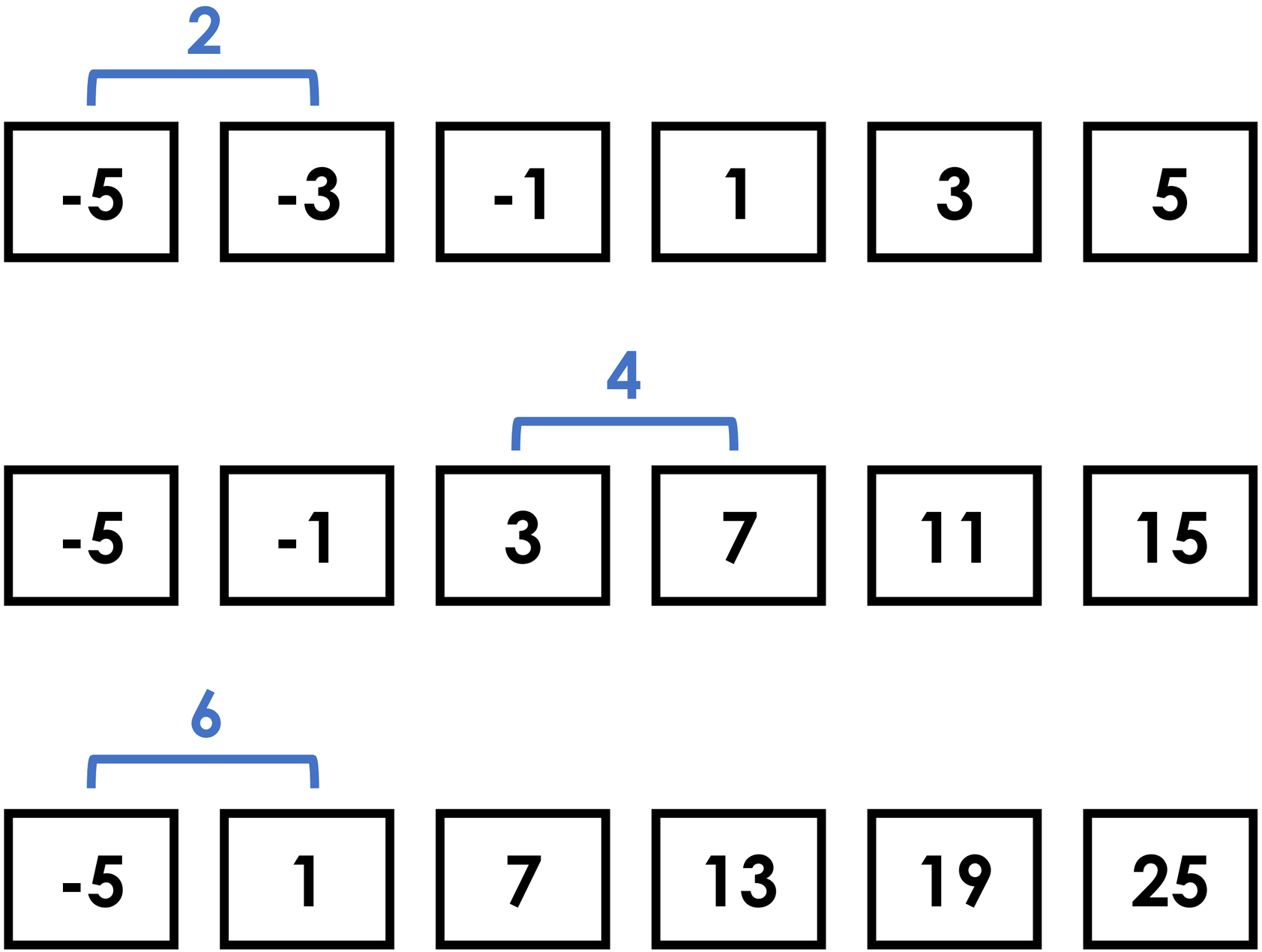

4
-5
3
11


2
-5
-3
-1
1
3
5


4
-5
-1
3
7
11
15


2
-5
-3
-1
1
3
5
4
-5
-1
3
7
11
15
12
-5
7


2
-5
-3
-1
1
3
5
4
-5
-1
3
7
11
15
6
-5
7


2
-5
-3
-1
1
3
5
4
-5
-1
3
7
11
15
6
-5
1
7
13
19
25

Small Difference Questions

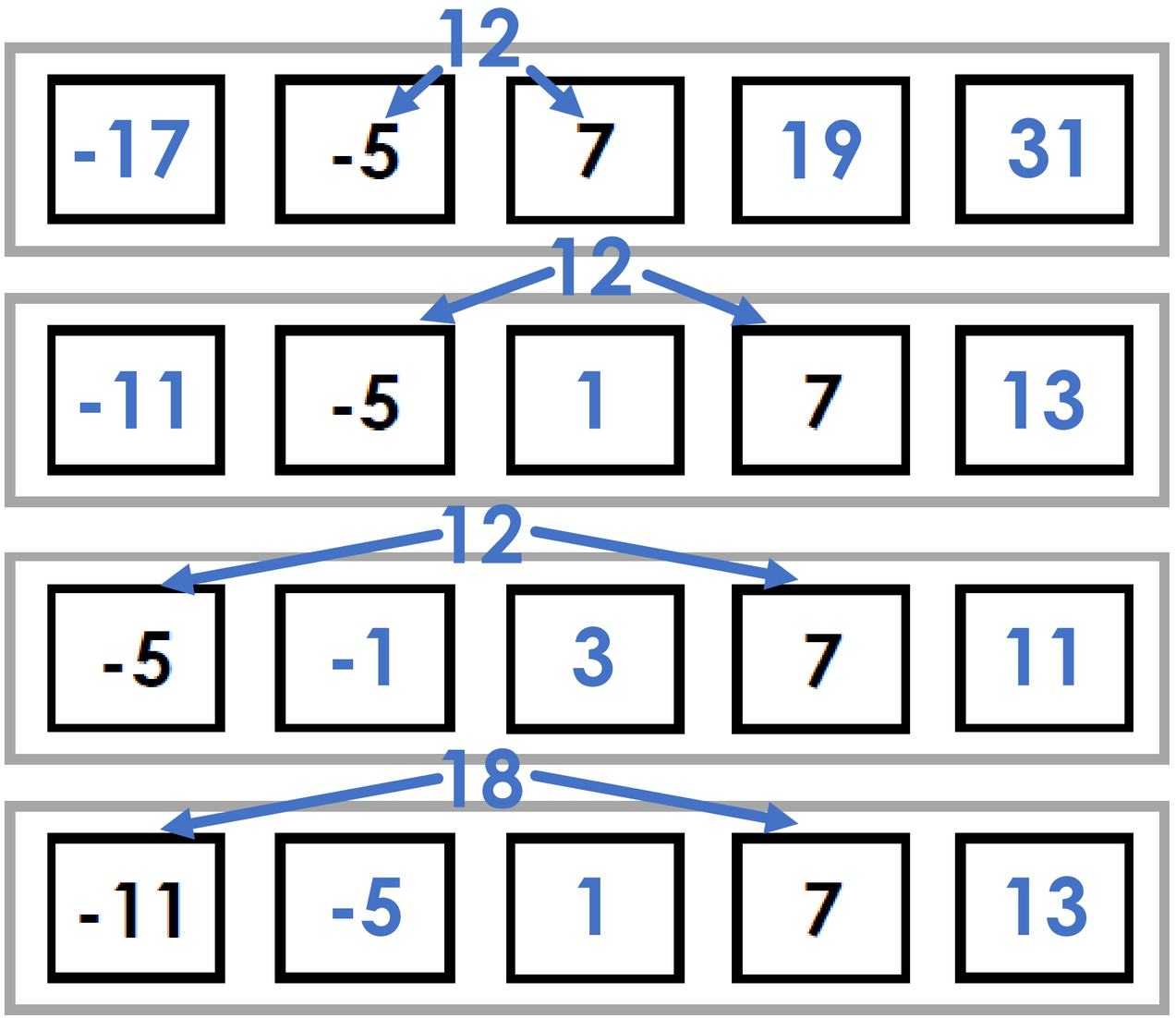
	-5	7		
	-5		7	
-5			7	
-11			7	

…is the same

…is different

Which sequence will have the largest 5th value?

Small Difference Questions



…is the same

…is different

Which sequence will have the largest 5th value?

Small Difference Questions

15 × 3 = 45	12 ÷ 3 = 4	30 ÷ 5 = 6
15 = 3 × 5	12 × 4 = 48	30 × 5 = 150
15 ÷ 3 = 5	48 ÷ 4 = 12	150 = 30 × 5
5 = 15 ÷ 3	4 = 12 ÷ 3	150 ÷ 5 = 30
45 ÷ 15 = 3	4 = 48 ÷ 12	30 = 150 ÷ 5

Extend: *design your own sequence of 5 multiplication and division questions using the same two numbers.*

20		×	4	=		
20		=	4	×		
20		÷	4	=		
		=	20	÷	4	
		÷	20	=	4	

Small Difference Questions

10 – 2 = 8

10 – 8 = 2

9 – 7 = 2

8 – 6 = 2

9 – 5 = 4

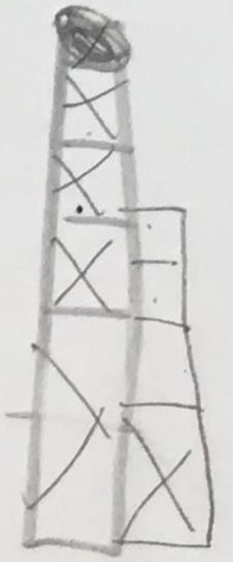
10 – 7 = 3

9 – 6 = 6

8 – 5 = 6

8 – 3 = 5

7 – 3 = 4



SUBTRACTION

5+5-1=9 4+5=9

I SEE N

Small Difference Questions

Finding or creating the sequences of questions

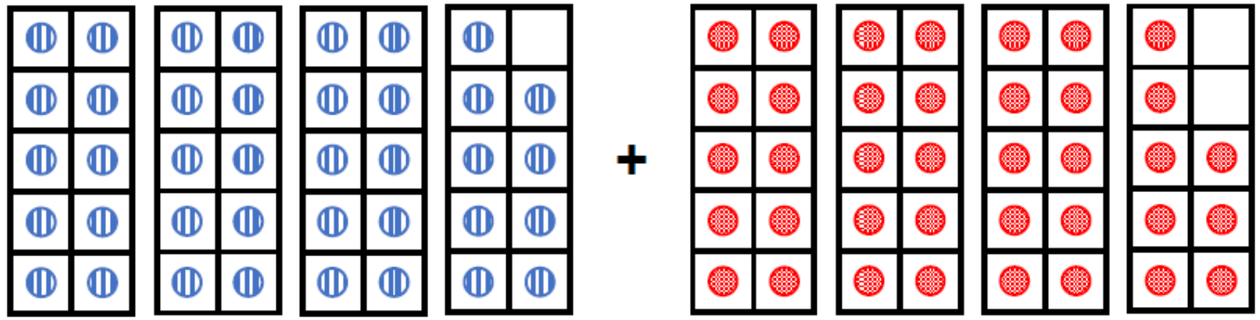
Introducing sequences in different year groups

Teaching children to spot similarities/differences, make predictions, explain relationships and create their own examples

Flexible

Different Methods
Rank by Difficulty

Different Methods



39 + 38 = ☐

Double ☐ subtract 3

Double 30 add ☐

40 + ☐

Different Ways

Ways to calculate 25 × 18:

50 × ☐

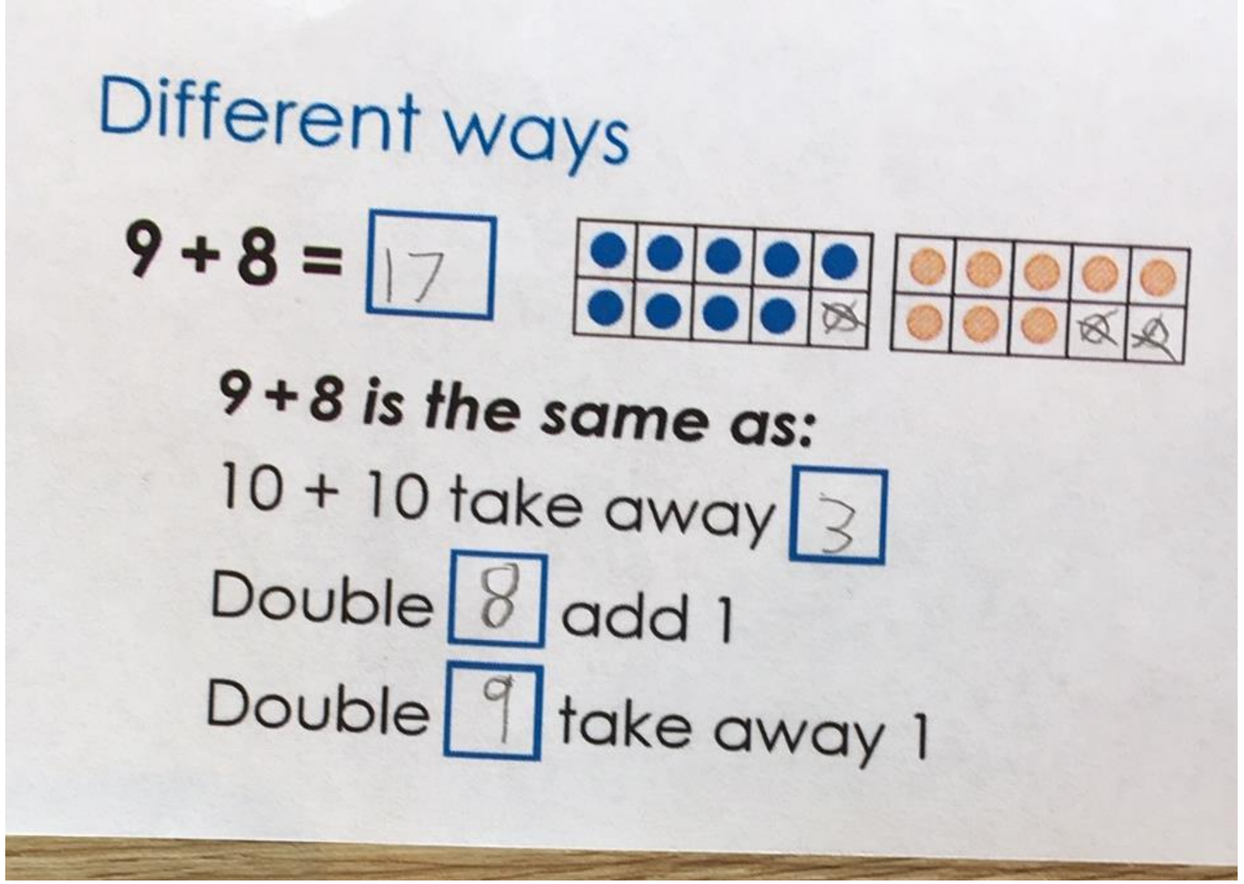
☐ less than 25 × 20

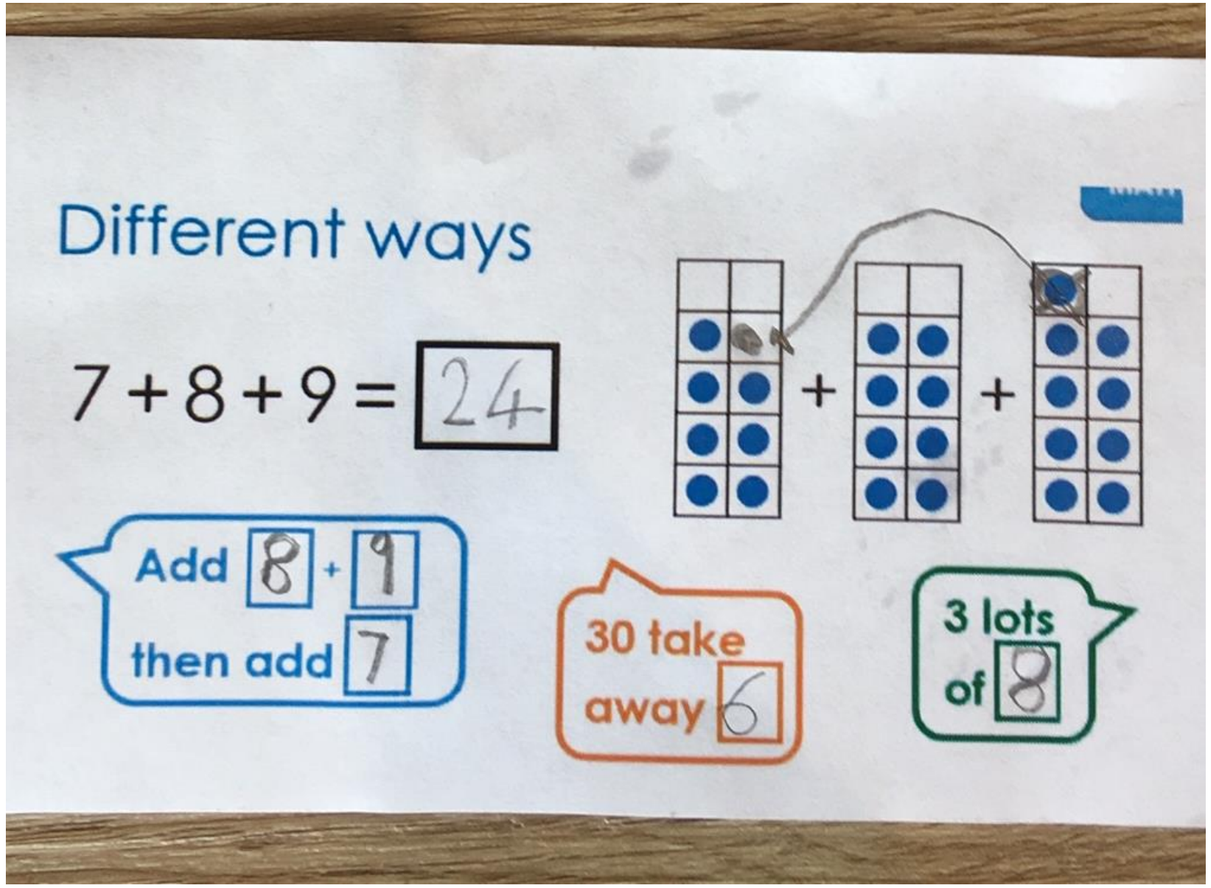
18 × ☐ × ☐

18 × ☐ ÷ 4

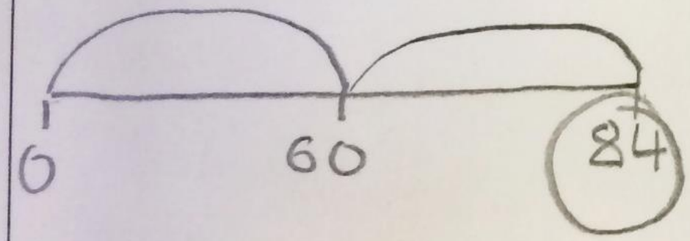
Flexible

Different Methods
Rank by Difficulty





12 x 7 = 84

Silver	Gold	Bronze
12×5=60 12×2=24  	6×7=42 42×2=84	11×7=77 1×7=7 77+7=84

Flexible

Different Methods
Rank by Difficulty

Rank by Difficulty

64% of 100 =

30% of 400 =

95% of 500 =

99% of 300 =

Did you use the same method for each calculation?

Rank by Difficulty

38 ÷ 4 = **27 ÷ 9 =** **40 ÷ 3 =**

80 ÷ 5 = **100 ÷ 4 =**

The answer to… has a remainder

I know the answer to… is a single-digit number because…

To find the answer to… I partitioned… into…

Rank by Difficulty

183 + 117 = **597 + 126 =** **370 + 280 =**

628 + 371 = **4050 + 602 =**

I answered… mentally by…

There are more steps to answer… because…

Small Difference Questions

299 + 100 = 399

299 – 100 = 199

299 – 10 = 289

299 + 10 = 309

698 + 100 = 798

698 + 10 = 708

107 – 100 = 7

107 – 10 = 97

The most difficult question(s) ***is/are****… because…*

Creative

Design questions

34 X 28

97 x 8

25 x 24

Ordered

How Many Ways?
Different Ways

How Many Ways?

Fill each gap with a **single-digit number:**

$$\frac{\square}{\square} \text{ of } 24 = \square$$

Level 1: *I can find an answer*

Level 2: *I can find different answers*

Level 3: *I know how many answers there are*

How Many Ways?

$$\begin{array}{r} 6\square 2 \\ -\ 2\ 3\square \\ \hline 3\square 5 \\ \hline \end{array}$$

Level 1: *I can find an answer.*

Level 2: *I can find different answers.*

Level 3: *I know how many answers there are.*

Extend: *Change one digit to create two more possible answers.*

Ordered

How Many Ways?
Different Ways

How Many Ways?

Fill each gap with a **single-digit number:**

$$\frac{\boxed{3}}{\boxed{8}} \text{ of } 24 = \boxed{9}$$

Level 1: *I can find an answer*

Level 2: *I can find different answers*

Level 3: *I know how many answers there are*

How Many Ways?

$$\begin{array}{r} 6\boxed{4}2 \\ -\ 2\ 3\boxed{7} \\ \hline 3\boxed{0}5 \\ \hline \end{array}$$

Level 1: *I can find an answer.*

Level 2: *I can find different answers.*

Level 3: *I know how many answers there are.*

Extend: *Change one digit to create two more possible answers.*

How Many Ways?

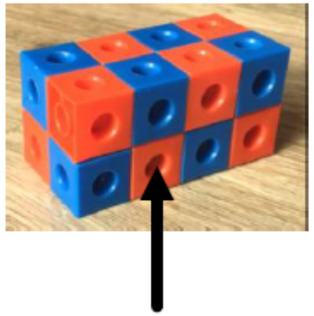
Make a cuboid using 24 to 30 cubes.

There must be at least 6 squares on each face of the cuboid.

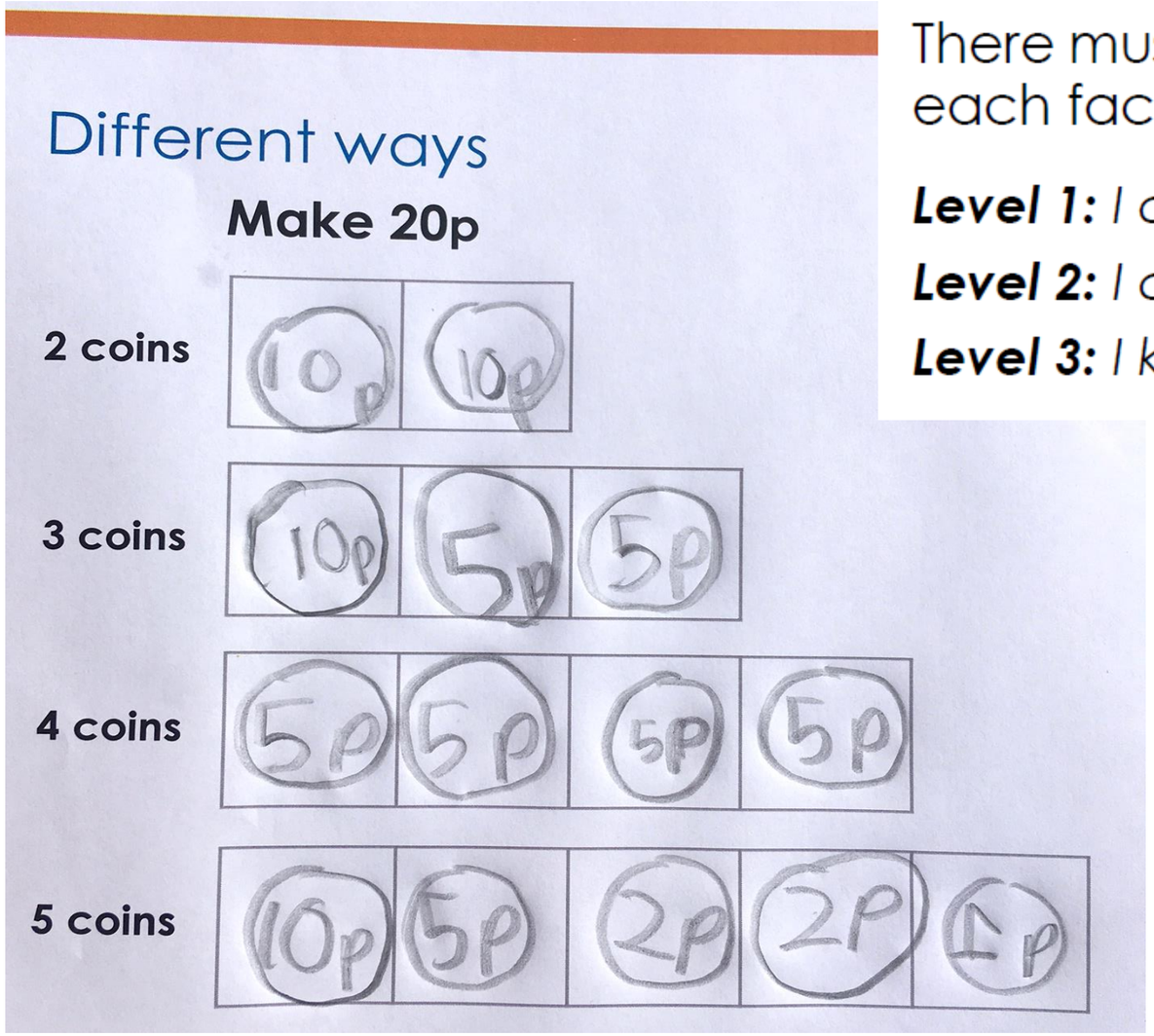
Level 1: *I can find a way*

Level 2: *I can find different ways*

Level 3: *I know how many ways there are*



8 squares on this face



Explicit learning behaviours

I start by myself *I tried… My idea was… I didn't know…*	
I follow the thinking of others *So you think… Why did you… I agree/disagree with…*	
I check my own success *I will do… by myself. I can easily… I need help to learn…*	
I show my understanding in different ways *Pictures Explanations Examples Explain mistakes*	
I can extend my thinking *…is similar to/different from… because… My example is…*	
Personal target:	

Teacher responses for consistent messages

Responding to a misunderstanding

'I need to… differently.' *'You can… next you need to…'*

Responding to errors

Less specific feedback? *Celebrating mistakes?*

Reflect the emotional experience of mathematics

Discomfort a precursor to achievement

Doubt at the point of answer

Recap

Classroom habits that protect limited attention

Using misconceptions and the power of predicting

Emphasising relationships between questions

Depth: comparing, creating and being systematic

Teacher responses: consistency and autonomy