## Enhancing Mathematical Reasoning in Y5 and Y6

Breaking down calculation
Using sequences of questions
Exploring word question structures
Flexible thinking, deep application


## Novice

## Expert

Start of

I know... so...
$72 \div 3=\mathbf{2 4}$
$78 \div 3=\square$
$98 \div 7=14$
$91 \div 7=\square$

$72 \div 4=18$
$144 \div 8=\square$
$48 \div 6=8$
$108 \div 6=\square$
$112 \div \mathbf{4}=\mathbf{2 8}$
$192 \div 4=\square$

I know... so...
$\mathbf{7 2 \div 3}=\mathbf{2 4}$
$78 \div 3=26$
$98 \div 7=14$
$91 \div 7=\square$

## $84 \div 6=14$ <br> $168 \div 6=28$

$72 \div 4=18$
$144 \div 8=\square$
$48 \div 6=8$
$108 \div 6=18$
$112 \div 4=28$
$192 \div 4=$

## Small Difference Questions

$56 \div 4=14$
$112 \div 4=28$
$224 \div 8=28$
$304 \div 8=38$
$344 \div 8=43$

## $108 \div 3=36$

$216 \div 6=$

$$
216 \div 3=\square
$$

$$
246 \div 3=\square
$$

$$
261 \div 3=
$$

$\square$

## Small Difference Questions

$56 \div 4=14$
$112 \div 4=28$
$224 \div 8=28$
$304 \div 8=38$
$344 \div 8=43$

## $108 \div \mathbf{3}=\mathbf{3 6}$

$216 \div 6=36$
$216 \div 3=\square$
$246 \div 3=\square$
$261 \div 3=$

## Small Difference Questions

$56 \div 4=14$
$112 \div 4=28$
$224 \div 8=28$
$304 \div 8=38$
$344 \div 8=43$

## $108 \div 3=36$

$$
216 \div 6=36
$$

$$
216 \div 3=72
$$

$$
246 \div 3=\square
$$

$$
261 \div 3=
$$

$\square$

## Small Difference Questions

$56 \div 4=14$
$112 \div 4=28$
$224 \div 8=28$
$304 \div 8=38$
$344 \div 8=43$

## $108 \div 3=36$

$$
216 \div 6=36
$$

$$
216 \div 3=72
$$

$$
246 \div 3=82
$$

$$
261 \div 3=
$$

$\square$

## Small Difference Questions

$56 \div 4=14$
$112 \div 4=28$
$224 \div 8=28$
$304 \div 8=38$
$344 \div 8=43$

## $108 \div 3=36$

$$
216 \div 6=36
$$

$$
216 \div 3=72
$$

$$
246 \div 3=82
$$

$$
261 \div 3=87
$$

## -Gralg bartonREFLECT, EXPECT, OHEGK, EXPLAIN <br> SEOUENCES AND BEHAVIOUR TO ENABLE

MATHEMATICAL THINKING IN THE CLASSROOM


## Reflect: spotting similarities/differences

Expect: how will the next answer be different?

> Check: was this what you expected? Seeing the answer you noticed...

Explain: this is because...

$$
\begin{aligned}
& 100-10 \times 6=\square \\
& 100-(10 \times 6)=\square \\
& (100-10) \times 6=\square \\
& (100-6) \times 10=\square \\
& 100-6 \times 10=\square \\
& 6 \times 10-100=\square
\end{aligned}
$$

$$
\begin{aligned}
& 100-10 \times 6=\square \\
& 100-(10 \times 6)=\square \\
& (100-10) \times 6=\square \\
& (100-6) \times 10=\square \\
& 100-6 \times 10=\square \\
& 6 \times 10-100=\square
\end{aligned}
$$

$$
\begin{aligned}
& 100-10 \times 6=\square 40 \\
& 100-(10 \times 6)=\square 40 \\
& (100-10) \times 6=\square \\
& (100-6) \times 10=\square \\
& 100-6 \times 10=\square \\
& 6 \times 10-100=\square
\end{aligned}
$$

$$
\begin{aligned}
& 100-10 \times 6=\boxed{40} \\
& 100-(10 \times 6)=\boxed{40} \\
& (100-10) \times 6=540 \\
& (100-6) \times 10=\square \\
& 100-6 \times 10=\square \\
& 6 \times 10-100=\square
\end{aligned}
$$

$$
\begin{aligned}
& 100-10 \times 6=40 \\
& 100-(10 \times 6)=40 \\
& (100-10) \times 6=540 \\
& (100-6) \times 10=940 \\
& 100-6 \times 10=\square \\
& 6 \times 10-100=\square
\end{aligned}
$$

$$
\begin{aligned}
& 100-10 \times 6=40 \\
& 100-(10 \times 6)=40 \\
& (100-10) \times 6=540 \\
& (100-6) \times 10=940 \\
& 100-6 \times 10=40 \\
& 6 \times 10-100=-40
\end{aligned}
$$

## Estimate

Position 0 on each number line:


## Estimate

Position 0 on each number line:


## Estimate

Position 0 on each number line:


## Estimate

Position 0 on each number line:


## Estimate

Position 0 on each number line:


## Estimate

Position 0 on each number line:


## Estimate

Position 0 on each number line:


## Estimate

Position 0 on each number line:


## Estimate

Position 0 on each number line:


## Small Difference Questions



## Small Difference Questions



1 more than XLIII is $X L \mid V$ 10 more than XLIII is $L\|\|$
10 less than XLIII is $X X X|I|$
1 less than $\mathbf{X X X}$ is


10 more than XXX is


50 more than XXX is

$\mathbf{X X X}$ is $\mathbf{3 0}$

$$
\begin{aligned}
& \mathrm{I}=1 \\
& \mathrm{~V}=5 \\
& \mathrm{X}=10 \\
& \mathrm{~L}=50
\end{aligned}
$$




## $385+146$

406-258
$18 \times 3.5$

# $385+146$ $400+131$ <br> 406-258 <br> 399-251 

$18 \times 3.5$
$9 \times 7$
$28+17$

## $28+17$ <br> $30+15$

## Simplify



Extend: Create your own 'simplify’ addition questions.

## Mental or Written Method?

$4731+5268=895+385=$
$463+278=$
$2480+2520=$

## $12-8=$

$12-8=4$|  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  | $\longleftrightarrow$ |  |  |  |



$$
13-9=
$$




$$
14-10=
$$

$$
12-8=4
$$



## Broken Calculator

'The 7 and 5 keys on my calculator are broken!' How can I use my calculator to work out:

$$
\begin{array}{ll}
750+850= & 7.5-5.5= \\
505-367= & 63.5+79.5= \\
866-597= & 1.4-0.7=
\end{array}
$$

Extend: design your own ‘Broken Calculator’ questions.

## $8 \times 3$

-०००००००
$\bullet \bullet \bullet \bullet \bullet \bullet \bullet$
$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$

## $8 \times 3$

0000000
-००००००
-००००००
$4 \times 6$

$10 \times 3$

$10 \times 3$

$10 \times 3$


$6 \times 5$


## Different Methods

Double one number, halve the other number, the product is the same.

$$
\text { Example: } 8 \times 12=96 \quad 16 \times 6=96
$$

Which of these questions are made easier by a doubling and halving strategy?
$26 \times 24$
$14 \times 25$
$1.5 \times 6$

## Broken Calculator

'The 2 and 5 keys on my calculator are broken!' How can I use my calculator to work out:
$35 \times 26$
$25 \times 12$

## Contexts

In these questions, some of the numbers are hidden. For each question, which operation is needed? addition + multiplication $\times$ division $\div$ (a) 10 friends spend a total of at the café. How much money does each person pay?
(b) Mike is 3 years old. Tom is years old.

When Mike is $\mathbf{9}$ years old, what age will Tom be?
(c) Joy has $\quad$ times as much money as Zara. Zara has $£ 12$. How much money does Joy have?

## Explore

At the market, apples cost 25 p each.
At the shop, it costs $£ 1.10$ for a bag of 6 apples.
Using this information, think of a question that involves:
(a) Multiplication
(b) Multiplication and addition
(c) Multiplication and subtraction
(d) Division

## Explore

1500 people are travelling from Sheffield to Leeds to go to the match. They travel by car or by coach. 200 people fit in a coach. 5 people fit in a car.
Using this information, think of a question with the answer:
(a) 6 coaches
(b) 140 cars

## Spot the Difference

Answer the questions:
(a) Ruth earns $£ 15$ each day delivering newspapers. How much money does Ruth earn each week?
(b) Kate plays the piano for 15 minutes every day. How long, in hours and minutes, does Kate spend playing the piano each week?

What's the same? What's different?

## Different Question Types

1. Amy bought a sandwich, a pizza slice and a drink. She paid £5. How much change does she get?
2. Tim has £5. He wants a sandwich, two pizza slices and a drink. How much more money does Tim need?
3. Sam has £10.

How many pizza slices can he afford?
4. Grace spent £ 10 on 5 items.

She got £2.85 change. What did she buy?

## Sandwich: £2.75 <br> Pizza Slice: £1.30 <br> Fruit: 45p <br> Drink: 60p

## Recap <br> Q\&A

Breaking down calculation
Using sequences of questions
Word questions: exploring structures
Flexible thinking, deep application

